

axiom™



The 30 Year Horizon

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Volume 4: Axiom Developers Guide

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New Foreword

On October 1, 2001 Axiom was withdrawn from the market and ended life as a commercial product. On September 3, 2002 Axiom was released under the Modified BSD license, including this document. On August 27, 2003 Axiom was released as free and open source software available for download from the Free Software Foundation's website, Savannah.

Work on Axiom has had the generous support of the Center for Algorithms and Interactive Scientific Computation (CAISS) at City College of New York. Special thanks go to Dr. Gilbert Baumslag for his support of the long term goal.

The online version of this documentation is roughly 1000 pages. In order to make printed versions we've broken it up into three volumes. The first volume is tutorial in nature. The second volume is for programmers. The third volume is reference material. We've also added a fourth volume for developers. All of these changes represent an experiment in print-on-demand delivery of documentation. Time will tell whether the experiment succeeded.

Axiom has been in existence for over thirty years. It is estimated to contain about three hundred man-years of research and has, as of September 3, 2003, 143 people listed in the credits. All of these people have contributed directly or indirectly to making Axiom available. Axiom is being passed to the next generation. I'm looking forward to future milestones.

With that in mind I've introduced the theme of the "30 year horizon". We must invent the tools that support the Computational Mathematician working 30 years from now. How will research be done when every bit of mathematical knowledge is online and instantly available? What happens when we scale Axiom by a factor of 100, giving us 1.1 million domains? How can we integrate theory with code? How will we integrate theorems and proofs of the mathematics with space-time complexity proofs and running code? What visualization tools are needed? How do we support the conceptual structures and semantics of mathematics in effective ways? How do we support results from the sciences? How do we teach the next generation to be effective Computational Mathematicians?

The "30 year horizon" is much nearer than it appears.

Tim Daly
CAISS, City College of New York
November 10, 2003 ((iHy))

Confronting every new programmer learning a new language are

- The Cave of Artifacts
- The Forest of Tooling
- The Mountain of Language
- The Cloud Castle of Mindset

– Daniel Higginbotham in Clojure for the Brave and True

0.1 What is the purpose of the HACKPI domain?

HACKPI is a hack provided for the benefit of the axiom interpreter. As a mathematical type, it is the simple transcendental extension $\mathbb{Q}(\pi)$ of the rational numbers. This type allows interactive users to use the name '`%pi`' without a type both where a numerical value is expected [as in `draw(sin x,x=-%pi..%pi)`] or when the exact symbolic value is meant. The interpreter defaults a typeless `%pi` to HACKPI and then uses the various conversions to cast it further as required by the context.

One could argue that it is unfair to single `%pi` out from other constants, but it occurs frequently enough in school examples (specially for graphs) so it was worth a special hack. In a non-interactive environment (library), HACKPI would not exist.

(Manuel Bronstein)

0.2 How Axiom Builds

The environment variables

Axiom uses a tree of Makefiles to build the system. Each Makefile is created from the literate file (Makefile.pamphlet) and then executed.

In order to have a complete set of variables we create an “environment” that contains all of the shell variables (except the AXIOM variable).

These can be changed on the command line at the time of the top level “make” command. One common usage pattern is to override the NOISE variable. This variable controls whether we see the full output or just the echo of each individual step. Sometimes a build fails at a step and we would like to know the details. By default they are written to `$TMP/trace` but we can watch every detail with the command line:

```
make NOISE=
```

This overrides the output file and writes everything to the console.

Another common usage pattern is to override the tests that are run. By default, all tests are run. This can be very time consuming. A particular subset can be run or, using the option “notests”, none will be run:

```
make TESTSET=notests
```

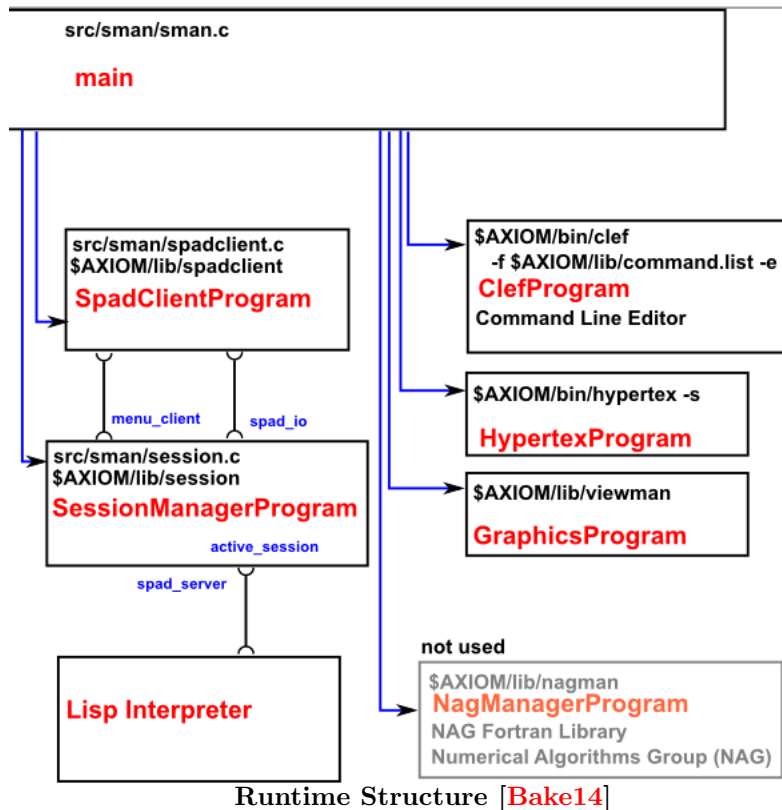
```
AWK=gawk
BOOKS=/research/test/books
BYE=bye
```

```

CC=gcc
CCF=-O2 -fno-strength-reduce -Wall -D_GNU_SOURCE -DLINUXplatform
-I/usr/X11/include
COMMAND=/usr/local/axiom/mnt/ubuntu/bin/axiom
DAASE=/research/test/src/share
DESTDIR=/usr/local/axiom
DOCUMENT=/research/test/mnt/ubuntu/bin/document
GCLDIR=/research/test/lsp/gcl-2.6.8pre4
GLOPTS=--enable-vssize=65536*2 --enable-locbfd --disable-dynsysbfd
--disable-statsysbfd --enable-maxpage=512*1024 --disable-xgcl
--disable-tkconfig
GCLVERSION=gcl-2.6.8pre4
INC=/research/test/src/include
INT=/research/test/int
LDF= -L/usr/X11R6/lib -L/usr/lib -lXpm
LISP=lsp
LSP=/research/test/lsp
MNT=/research/test/mnt
NOISE=-o /research/test/obj/tmp/trace
O=o
OBJ=/research/test/obj
PART=cprogs
PATCH=patch
PLF=LINUXplatform
RANLIB=ranlib
RUNTYPE=serial
SPAD=/research/test/mnt/
SPADBIN=/research/test/mnt/ubuntu/bin
SPD=/research/test
SRC=/research/test/src
SRCDIRS=interpdir sharedir algebradir etcdir clefdir docdir graphdir
smadir hyperdir browserdir inputdir
SUBPART=everything
SYS=ubuntu
TANGLE=/research/test/mnt/ubuntu/bin/lib/notangle
TAR=tar
TESTSET=none
TMP=/research/test/obj/tmp
TOUCH=touch
UNCOMPRESS=gunzip
VERSION=Axiom (May 2010)
WEAVE=/research/test/mnt/ubuntu/bin/lib/noweave
XLIB=/usr/X11R6/lib
ZIPS=/research/test/zips

```

0.3 The runtime structure of Axiom



The build step

This shows the steps taken to build Axiom in the sequence they happen. Each level of indentation is another level of Makefile being executed.

Makefile

```

1 noweb
2 copy SRC/scripts to AXIOM/bin
3 extract Makefile.SYS from Makefile.pamphlet
4 latex SRC/input/*.input.pamphlet
5 extract SRC/algebra/Makefile.help from SRC/algebra/Makefile.pamphlet
5a make SRC/algebra/Makefile.help parallelhelp
5a1 extract syntax help from BOOKS/bookvol15
5a2 extract help files from BOOKS/bookvol10.*
6 extract BOOKS/Makefile from BOOKS/Makefile.pamphlet
6a make BOOKS/Makefile
6a1 copy SRC/scripts/tex/axiom.sty to AXIOM/doc
6a2 create AXIOM/doc/*.pdf
6a2a copy book*.pamphlet to AXIOM/doc
6a2b extract latex for each book
6a2c latex each book
  
```

```

        6a2d dvipdfm each dvi file
        6a3 make AXIOM/doc/toc.pdf
7 extract AXIOM/doc/hypertex/Makefile1 from BOOKS/bookvol11
  7a make AXIOM/doc/hypertex/Makefile1
    7a1 extract all xhtml pages to AXIOM/doc/hypertex
    7a2 extract axiom1.bitmap from BOOK/bookvol11
    7a3 extract rcm3720.input from BOOK/bookvol11
    7a4 extract strang.input from BOOK/bookvol11
    7a5 extract signatures.txt from BOOK/bookvol11
    7a6 copy BOOKS/ps/doctitle.png to AXIOM/doc/hypertex
    7a7 copy BOOKS/ps/lightbayou.png to AXIOM/doc/hypertex
8 make Makefile.SYS
  8a create the root directories
  8b create noweb if needed
  8c extract SRC/Makefile from SRC/Makefile.pamphlet
  8d make SRC/Makefile setup
    8d1 extract SRC/scripts/Makefile from SRC/scripts/Makefile.pamphlet
    8d2 make SRC/scripts/Makefile
      8d1a copy all scripts to AXIOM/bin
    8d3 extract SRC/lib/Makefile from SRC/lib/Makefile.pamphlet
    8d4 make SRC/lib/Makefile
      8d4a compile INT/lib/bsdsignal.c
      8d4b compile INT/lib/cursor.c
      8d4c compile INT/lib/edin.c
      8d4d compile INT/lib/fnct-key.c
      8d4e compile INT/lib/halloc.c
      8d4f compile INT/lib/openpty.c
      8d4g compile INT/lib/pixmap.c
      8d4h compile INT/lib/prt.c
      8d4i compile INT/lib/sockio-c.c
      8d4j compile INT/lib/spadcolors.c
      8d4k compile INT/lib/util.c
      8d4l compile INT/lib/wct.c
      8d4m compile INT/lib/xdither.c
      8d4n compile INT/lib/xshade.c
      8d4o compile INT/lib/xspadfill.c
      8d4p create libspad.a
      8d4q compile INT/lib/cfuncs-c.c
      8d4r compile INT/lib/hash.c
      8d4s latex all files to INT/doc/src/lib
  8e extract LSP/Makefile from LSP/Makefile.pamphlet
  8f make LSP/Makefile gclmdir
    8f1 untar ZIPS/gcl
    8f2 apply Axiom patches to gcl
    8f3 copy gcl_collectfn.lsp to OBJ/SYS/lsp
    8f4 copy sys-proclaim.lisp to OBJ/SYS/lsp
    8f5 make LSP/GCLVERSION/Makefile
    8f6 add BOOKS/tangle.lsp to gcl to create INT/SYS/lisp
  8g make SRC/Makefile
    8g1 make stanzas from SRCDIRS
      8g1a interpdir
        8g1a1 copy bookvol5 to src/interp
        8g1a2 copy bookvol9 to src/interp
        8g1a3 copy bookvol10.5 to src/interp

```

```

8g1a4 extract util.ht from BOOKS/bookvol7.1 to AXIOM/doc
8g1a5 make SRC/interp/Makefile
    8g1a5a build SAVESYS=OBJ/SYS/bin/interpsys
        8g1a5a1 build DEPSYS=OBJ/SYS/bin/depsys
        8g1a5a2 compile all interp files
        8g1a5a3 call build-interpsys to make SAVESYS
        8g1a5a4 build warm.data
        8g1a5a5 build SAVESYS
        8g1a5a6 copy SAVESYS to AXIOMSYS=AXIOM/bin/AXIOMsys
8g1b sharedir
    8g1b1 make share/Makefile
        8g1b1a copy SRC/share/algebra/command.list to AXIOM/lib
8g1c algebradir
    8g1c1 extract algebra/Makefile from SRC/algebra/Makefile.pamphlet
    8g1c2 copy bookvol10.2 to SRC/algebra
    8g1c3 copy bookvol10.3 to SRC/algebra
    8g1c4 copy bookvol10.4 to SRC/algebra
    8g1c5 copy bookvol10.5 to SRC/algebra
    8g1c6 extract 'findAlgebraFiles'
        from SRC/algebra/Makefile.pamphlet
    8g1c7 execute findAlgebraFiles and append output
        to SRC/algebra/Makefile
    8g1c8 make SRC/algebra/Makefile
        8g1c8a build INT/algebra nrlibs
        8g1c8b copy SRC/algebra/libdb.text to AXIOM/algebra
        8g1c8c construct AXIOM/bin/index.html
        8g1c8d copy SRC/share/algebra/gloss.text AXIOM/algebra
        8g1c8e copy SRC/share/algebra/glossdef.text AXIOM/algebra
        8g1c8f copy SRC/share/algebra/glosskey.text AXIOM/algebra
8g1d etcdir
    8g1d1 extract SRC/etc/Makefile from SRC/etc/Makefile.pamphlet
    8g1d2 make etc/Makefile
        8g1d2a copy SRC/doc/gloss.text INT/algebra
        8g1d2b copy SRC/doc/topics.data INT/algebra
        8g1d2c call make-databases
        8g1d2b copy INT/algebra/*.daase AXIOM/algebra
        8g1d2e compile asq.c
        8g1d2f copy OBJ/SYS/etc/asq AXIOM/bin
        8g1d2g copy SRC/etc/summary AXIOM/lib
        8g1d2h copy SRC/etc/copyright AXIOM/lib
8g1e clefdir
    8g1e1 extract SRC/clef/Makefile from SRC/clef/Makefile.pamphlet
    8g1e2 make clef/Makefile
        8g1e2a extract edible.c to OBJ/SYS/clef
        8g1e2b compile OBJ/SYS/clef/edible.c
        8g1e2c link edible, fnct-key, edin, bsdsignal, prt, wct,
            openpty, cursor into AXIOM/bin/clef
8g1f docdir
    8g1f1 extract SRC/doc/Makefile from SRC/doc/Makefile.pamphlet
    8g1f2 make SRC/doc/Makefile
        8g1f2a extract SRC/doc/axiom.bib to INT/doc
        8g1f2b extract SRC/doc/axiom.sty to AXIOM/bin/tex
        8g1f2c extract SRC/doc/refcard.dvi to AXIOM/doc
        8g1f2d extract SRC/doc/endorpaper.dvi to AXIOM/doc

```

```

      8g1f2e copy SRC/doc/ps/* to AXIOM/doc/ps
      8g1f2f extract SRC/doc/rosetta.dvi to AXIOM/doc
8g1f2g extract SRC/doc/booklet.c to INT
      8g1f2h compile booklet.c
      8g1f2i copy booklet to AXIOM/bin
8g1g graphdir
      8g1g1 extract SRC/graph/Makefile from BOOKS/bookvol8.pamphlet
      8g1g2 make graph/Makefile
          8g1g2a compile and link AXIOM/lib/viewman
          8g1g2b compile and link AXIOM/lib/view2d
          8g1g2c compile and link AXIOM/lib/view3d
          8g1g2d compile and link AXIOM/lib/viewalone
          8g1g2e extract AXIOM/graph/parabola.view from bookvol8
          8g1g2f extract psfiles from bookvol8 to AXIOM/lib/graph
8g1h smandir
      8g1h1 extract SRC/sman/Makefile from BOOKS/bookvol6.pamphlet
      8g1h2 make sman/Makefile
          8g1h2a extract INT/sman/session.c from bookvol6
          8g1h2b compile INT/sman/session.c to OBJ/SYS/sman/session.o
          8g1h2c link OBJ/SYS/sman/session.o to AXIOM/lib/session
          8g1h2d extract INT/sman/spadclient.c from bookvol6
          8g1h2e compile INT/sman/spadclient.c
              to OBJ/SYS/sman/spadclient.o
          8g1h2f link OBJ/SYS/sman/spadclient.o to AXIOM/lib/spadclient
          8g1h2g extract INT/sman/sman.c from bookvol6
          8g1h2h compile INT/sman/sman.c to OBJ/SYS/sman/sman.o
          8g1h2i link OBJ/SYS/sman/sman.o to AXIOM/lib/sman
          8g1h2j extract axiom shell script from bookvol6 to AXIOM/bin
          8g1h2k chmod axiom shell script to be executable
          8g1h2l create AXIOM/doc/bookvol6.dvi
8g1i hyperdir
      8g1i1 extract INT/hyper/Makefile from BOOKS/bookvol7.pamphlet
      8g1i2 make INT/hyper/Makefile (to make hyperdoc)
          8g1i2a extract and compile AXIOM/lib/spadbuf
          8g1i2b extract and compile AXIOM/lib/ex2ht
          8g1i2c extract and compile AXIOM/bin/htadd
          8g1i2d extract and compile AXIOM/lib/hthits
          8g1i2e extract and compile AXIOM/bin/htsearch
          8g1i2f extract and compile AXIOM/lib/presea
          8g1i2g extract and compile AXIOM/bin/hypertex
          8g1i2h untar SPD/books/axbook.tgz to AXIOM/doc
          8g1i2j copy SPD/books/bigbayou.png to AXIOM/doc
          8g1i2k copy SPD/books/doctitle.png to AXIOM/doc
      8g1i3 extract INT/hyper/Makefile from BOOKS/bookvol7.1.pamphlet
      8g1i4 make INT/hyper/Makefile (to make hyperdoc pages)
          8g1i4a copy SPD/books/bookvol7.1 to AXIOM/doc
          8g1i4b htadd pages from AXIOM/doc/bookvol7.1
          8g1i4c copy SPD/books/bitmaps AXIOM/doc/bitmaps
          8g1i4d copy SPD/books/viewports AXIOM/doc/viewports
          8g1i4e untar AXIOM/doc/viewports .Z files
8g1j browserdir
      8g1j1 build of hyperdoc browser commented out
8g1k inputdir
      8g1k1 extract SRC/input/Makefile from SRC/input/Makefile.pamphlet

```

```

8g1k2 make SRC/input/Makefile
8g1k2a copy SRC/input/*.input INT/input
8g1k2b lisp tangle input files from SRC/input/*.input.pamphlet
8g1k2c extract INT/input/Makefile
      from SRC/input/Makefile.pamphlet
8g1k2d make INT/input/Makefile TESTSET
8g1k2d1 run regresstests
8g1k2d2 run catstests
8g1k2d3 run richtests
8g1k2d4 run regression tests
8g1k2d5 extract INT/input/Makefile.algebra
      from SRC/algebra/Makefile.pamphlet
8g1k2d6 make INT/input/Makefile.algebra

```

Where each output file is created

Here we show which step in the above set of actions creates the file that ends up in the final ship directory. We break it down by subdirectory in the final image.

AXIOM/algebra

in AXIOM/algebra:

```

*.o
browse.daase
category.daase
compress.daase
dependents.daase
interp.daase
operation.daase
users.daase

```

AXIOM/autoload

in AXIOM/autoload:

```

ax.o
bc-matrix.o
br-con.o
ht-util.o
mark.fn
mark.o
nag-c02.o
nag-c05.o
nag-c06.o
nag-d01.o
nag-d02.o
nag-d03.o
nag-e01.o
nag-e02.o
nag-e04.o
nag-f01.o

```


nag-f02.o
nag-f04.o
nag-f07.o
nag-s.o
nspadaux.o
pspad1.fn
pspad1.o
pspad2.fn
pspad2.o
topics.o
wi1.fn
wi1.o
wi2.fn
wi2.o

AXIOM/bin

in AXIOM/bin:

asq	8g1d2f	copy OBJ/SYS/etc/asq AXIOM/bin
axiom	8g1h2k	chmod axiom shell script to be executable
axiom.sty	6a1	copy SRC/scripts/tex/axiom.sty to AXIOM/doc
AXIOMsys	8g1a5a6	copy SAVESYS to AXIOMSYS=AXIOM/bin/AXIOMsys
booklet	8g1f2i	copy booklet to AXIOM/bin
boxhead	2	copy SRC/scripts to AXIOM/bin
boxtail	2	copy SRC/scripts to AXIOM/bin
boxup	2	copy SRC/scripts to AXIOM/bin
clef	8g1e2c	link edible, fnct-key, edin, bsdsignal, prt, wct, openpty, cursor into AXIOM/bin/clef
document	2	copy SRC/scripts to AXIOM/bin
htadd	8g1i2c	extract and compile AXIOM/bin/htadd
htsearch	8g1i2e	extract and compile AXIOM/bin/htsearch
hypertex	8g1i2g	extract and compile AXIOM/bin/hypertex
index.html	8g1c8c	construct AXIOM/bin/index.html
lib	1	noweb
btdefn		
cpif		
emptydefn		
finduses		
h2a		
htmldoc		
markup		
mnt		
nodefs		
noidx		
noindex		
noroff		
noroots		
notangle		
nountangle		
noweave		
noweb		
nt		

```

nuweb2noweb
numtime
pipedocs
tmac.w
toascii
tohtml
toroff
totex
unmarkup
Makefile.pamphlet
man
  man1
    cpif.1
    htmltoc.1
    nodefs.1
    noindex.1
    noroff.1
    noroots.1
    notangle.1
    nountangle.1
    noweave.1
    noweb.1
    nuweb2noweb.1
    sl2h.1
  man7
    nowebfilters.7
    nowebstyle.7
showdvi      2      copy SRC/scripts to AXIOM/bin
? sman      8g1h2i link OBJ/SYS/sman/sman.o to AXIOM/lib/sman
SPADEDIT     2      copy SRC/scripts to AXIOM/bin
tex          2      copy SRC/scripts to AXIOM/bin
  axiom.sty  8g1f2b extract SRC/doc/axiom.sty to AXIOM/bin/tex
            2      copy SRC/scripts to AXIOM/bin
  noweb.sty  1      noweb
  nwmac.tex  1      noweb
? viewalone  8g1g2d compile and link AXIOM/lib/viewalone

```

AXIOM/doc

```

AXIOM/doc:
axbook
  *.xhtml
axiom.sty
bigbayou.png
bitmaps
  *.bitmap
bookvol0.out
bookvol0.pdf
bookvol0.toc
bookvol10.1.out
bookvol10.1.pdf
bookvol10.1.toc

```

bookvol10.2.out
bookvol10.2.pdf
bookvol10.2.toc
bookvol10.3.out
bookvol10.3.pdf
bookvol10.3.toc
bookvol10.4.out
bookvol10.4.pdf
bookvol10.4.toc
bookvol10.5.out
bookvol10.5.pdf
bookvol10.5.toc
bookvol10.out
bookvol10.pdf
bookvol10.toc
bookvol11.out
bookvol11.pdf
bookvol11.toc
bookvol12.out
bookvol12.pdf
bookvol12.toc
bookvol1.out
bookvol1.pdf
bookvol1.toc
bookvol2.out
bookvol2.pdf
bookvol2.toc
bookvol3.out
bookvol3.pdf
bookvol3.toc
bookvol4.out
bookvol4.pdf
bookvol4.toc
bookvol5.out
bookvol5.pdf
bookvol5.toc
bookvol6.out
bookvol6.pdf
bookvol6.toc
bookvol7.out
bookvol7.pdf
bookvol7.toc
bookvol7.1.out
bookvol7.1.pamphlet
bookvol7.1.pdf
bookvol7.1.toc
bookvol8.out
bookvol8.pdf
bookvol8.toc
bookvol9.out
bookvol9.pdf
bookvol9.toc
bookvolbib.pdf
doctitle.png

```

endpaper.dvi
ht.db
hypertex
  *.xhtml
msgs
  s2-us.msgs
ps
  *.ps
refcard.dvi
rosetta.dvi
spadhelp
  *.help
src
? algebra
  algebra.Makefile.dvi
  books.Makefile.dvi
  clef
    axiom.sty
    edible.c.dvi
  clef.Makefile.dvi
  doc.Makefile.dvi
  etc.Makefile.dvi
? hyper
  input
    *.input.dvi
  input.Makefile.dvi
? interp
  interp.Makefile.dvi
lib
  *.c.dvi
  lib.Makefile.dvi
  Makefile.dvi
  root.Makefile.dvi
  scripts.Makefile.dvi
  share.Makefile.dvi
? sman
  src.Makefile.dvi
toc.pdf
util.ht
viewports
  *.view
  data
  graph0
  image.bm
  image.xpm

```

AXIOM/graph

```

AXIOM/graph
  parabola.view:      8g1g2e extract AXIOM/graph/parabola.view from bookvol8
  data
  graph0

```

AXIOM/input

AXIOM/input:
*.input files

AXIOM/lib

AXIOM/lib:

command.list	8g1b1a copy SRC/share/algebra/command.list to AXIOM/lib
copyright	8g1d2h copy SRC/etc/copyright AXIOM/lib
ex2ht	8g1i2b extract and compile AXIOM/lib/ex2ht
graph	8g1g2f extract psfiles from bookvol8 to AXIOM/lib/graph
colorpoly.ps	
colorwol.ps	
drawarc.ps	
drawcolor.ps	
drawIstr.ps	
drawline.ps	
drawlines.ps	
drawpoint.ps	
draw.ps	
drawrect.ps	
drawstr.ps	
drwfilled.ps	
end.ps	
fillarc.ps	
fillpoly.ps	
fillwol.ps	
header.ps	
setup.ps	
hthits	8g1i2d extract and compile AXIOM/lib/hthits
presea	8g1i2f extract and compile AXIOM/lib/presea
session	8g1h2c link OBJ/SYS/sman/session.o to AXIOM/lib/session
spadbuf	8g1i2a extract and compile AXIOM/lib/spadbuf
spadclient	8g1h2f link OBJ/SYS/sman/spadclient.o AXIOM/lib/spadclient
SPAEDIT	
summary	8g1d2g copy SRC/etc/summary AXIOM/lib
view2d	8g1g2b compile and link AXIOM/lib/view2d
view3d	8g1g2c compile and link AXIOM/lib/view3d
viewman	8g1g2a compile and link AXIOM/lib/viewman

AXIOM/src

AXIOM/src:
? algebra

AXIOM/timestamp

AXIOM/timestamp

0.4 How Axiom Works**Input and Type Selection**

First we change the default setting for autoload messages to turn off the noise of file loading from the library:

```
(1) -> )set mes auto off
```

Next we tell the interpreter to show us the modemap used to classify input and select types. This is known as “bottomup” messages. We can watch the interpreter ponder the input.

```
(1) -> )set mes bot on
```

Now we give it something nontrivial to ponder.

```
(1) -> f:=1/(a*x+b)
```

After parsing the input Axiom begins to figure out the type of the expression. In this case it starts with the multiply operator in the denominator.

Axiom has determined that “a” is of type VARIABLE and “x” is of type VARIABLE. It is looking for function of the form

```
VARIABLE * VARIABLE
```

so it looks in the domain of the left argument “a” which is VARIABLE and does not find the required function. Similarly it looks in the domain of the right argument “x” which is VARIABLE and, not surprisingly, does not find the required function.

It tried to promote each VARIABLE to SYMBOL and looks for a way to multiply VARIABLES and SYMBOLS or SYMBOLS and SYMBOLS. Neither succeeds.

```
Function Selection for *
```

```
Arguments: (VARIABLE a,VARIABLE x)
```

```
-> no appropriate * found in Variable a
-> no appropriate * found in Variable x
-> no appropriate * found in Symbol
-> no appropriate * found in Variable a
-> no appropriate * found in Variable x
-> no appropriate * found in Symbol
```

```
Modemaps from Associated Packages
```

```
no modemaps
```

Since it cannot find a specific modemap that uses the exact types it now expands the search to look for the general modemap. It searches these modemap in order to try to find one that fits.

```
Remaining General Modemaps
```

```
[1] (D,D1) -> D from D
    if D has XFALG(D2,D1) and D2 has ORDSET and D1 has RING
```

The first match will fail because Symbol does not have RING. We can determine this by asking the interpreter:

SYMBOL has RING

(1) false

Type: Boolean

The following modemap will fail for various similar reasons:

- [2] (D1,D) -> D from D
if D has XFALG(D1,D2) and D1 has ORDSET and D2 has RING
- [3] (Integer,D) -> D from D
if D has VECTCAT D2 and D2 has TYPE and D2 has ABELGRP
- [4] (D1,D) -> D from D
if D has VECTCAT D1 and D1 has TYPE and D1 has MONOID
- [5] (D,D1) -> D from D
if D has VECTCAT D1 and D1 has TYPE and D1 has MONOID
- [6] (D,D1) -> D1 from D
if D has SMATCAT(D2,D3,D4,D1) and D3 has RING and D4 has
DIRPCAT(D2,D3) and D1 has DIRPCAT(D2,D3)
- [7] (D1,D) -> D1 from D
if D has SMATCAT(D2,D3,D1,D4) and D3 has RING and D1 has
DIRPCAT(D2,D3) and D4 has DIRPCAT(D2,D3)
- [8] (D,D) -> D from D if D has SGROUP
- [9] (D,D1) -> D from D if D has RMODULE D1 and D1 has RNG
- [10] (D,D) -> D from D if D has MONAD
- [11] (D,D) -> D from D
if D has MATCAT(D1,D2,D3) and D1 has RING and D2 has FLAGG
D1 and D3 has FLAGG D1
- [12] (D1,D) -> D from D
if D has MATCAT(D1,D2,D3) and D1 has RING and D2 has FLAGG
D1 and D3 has FLAGG D1
- [13] (D,D1) -> D from D
if D has MATCAT(D1,D2,D3) and D1 has RING and D2 has FLAGG
D1 and D3 has FLAGG D1
- [14] (Integer,D) -> D from D
if D has MATCAT(D2,D3,D4) and D2 has RING and D3 has FLAGG
D2 and D4 has FLAGG D2
- [15] (D,D1) -> D1 from D
if D has MATCAT(D2,D3,D1) and D2 has RING and D3 has FLAGG
D2 and D1 has FLAGG D2
- [16] (D1,D) -> D1 from D
if D has MATCAT(D2,D1,D3) and D2 has RING and D1 has FLAGG
D2 and D3 has FLAGG D2
- [17] ((D5 -> D6),(D4 -> D5)) -> (D4 -> D6) from MappingPackage3(D4,
D5,D6)
if D4 has SETCAT and D5 has SETCAT and D6 has SETCAT
- [18] (D1,D) -> D from D if D has LMODULE D1 and D1 has RNG
- [19] (PolynomialIdeals(D1,D2,D3,D4),PolynomialIdeals(D1,D2,D3,D4))
-> PolynomialIdeals(D1,D2,D3,D4)
from PolynomialIdeals(D1,D2,D3,D4)
if D1 has FIELD and D2 has OAMONS and D3 has ORDSET and D4
has POLYCAT(D1,D2,D3)
- [20] (D1,D) -> D from D
if D has GRMOD(D1,D2) and D1 has COMRING and D2 has ABELMON

```

[21] (D,D1) -> D from D
      if D has GRMOD(D1,D2) and D1 has COMRING and D2 has ABELMON

[22] (D1,D2) -> D from D
      if D has FMCAT(D1,D2) and D1 has RING and D2 has SETCAT

[23] (D1,D2) -> D from D
      if D has FAMONC(D2,D1) and D2 has SETCAT and D1 has CABMON

[24] (Equation D1,D1) -> Equation D1 from Equation D1
      if D1 has SGROUP and D1 has TYPE

[25] (D1,Equation D1) -> Equation D1 from Equation D1
      if D1 has SGROUP and D1 has TYPE

[26] (D,D1) -> D from D
      if D has DIRPCAT(D2,D1) and D1 has TYPE and D1 has MONOID

[27] (D1,D) -> D from D
      if D has DIRPCAT(D2,D1) and D1 has TYPE and D1 has MONOID

[28] (DenavitHartenbergMatrix D2,Point D2) -> Point D2
      from DenavitHartenbergMatrix D2
      if D2 has Join(Field,TranscendentalFunctionCategory)

[29] (PositiveInteger,Color) -> Color from Color
[30] (DoubleFloat,Color) -> Color from Color
[31] (CartesianTensor(D1,D2,D3),CartesianTensor(D1,D2,D3)) ->
      CartesianTensor(D1,D2,D3)
      from CartesianTensor(D1,D2,D3)
      if D1: INT and D2: NNI and D3 has COMRING

[32] (PositiveInteger,D) -> D from D if D has ABELSG
[33] (NonNegativeInteger,D) -> D from D if D has ABELMON
[34] (Integer,D) -> D from D if D has ABELGRP

```

Eventually the interpreter decides that it can coerce Symbol to Polynomial(Integer). We can do this in the interpreter also:

```
a::Symbol::POLY(INT)
```

```
(1) a
```

```
      Type: Polynomial Integer
```

And the interpreter can find multiply in POLY(INT):

```

[1] signature: (POLY INT,POLY INT) -> POLY INT
      implemented: slot $$$ from POLY INT
[2] signature: (POLY INT,POLY INT) -> POLY INT
      implemented: slot $$$ from POLY INT

```

We can see this signature exists by asking the interpreter to show us the domain POLY(INT) (truncated here for brevity):

```

)show POLY(INT)
Polynomial Integer is a domain constructor.
Abbreviation for Polynomial is POLY
This constructor is exposed in this frame.

```


Issue)edit src/algebra/POLY.spad to see algebra source code for POLY

----- Operations -----

```

???: (Fraction Integer,%) -> %      ???: (Integer,%) -> %
???: (PositiveInteger,%) -> %      ???: (%,Fraction Integer) -> %
???: (%,Integer) -> %              ???: (%,%) -> %

```

Having found multiply the interpreter now starts a search for the operation

(POLY(INT)) + (VARIABLE)

It cannot find this modemap

```

Function Selection for +
  Arguments: (POLY INT,VARIABLE b)
-> no appropriate + found in Polynomial Integer
-> no appropriate + found in Variable b
-> no appropriate + found in Variable b

```

so it promotes VARIABLE to POLY(INT) and finds the operation:

(POLY(INT)) + (POLY(INT))

```

[1] signature: (POLY INT,POLY INT) -> POLY INT
    implemented: slot $$$ from POLY INT

```

Next it tackles the division operation where the numerator is PI (PositiveInteger) and the denominator is POLY(INT). It tries to find

(PI) / (POLY(INT))

in PositiveInteger, Polynomial Integer and Integer. All attempts fail.

```

Function Selection for /
  Arguments: (PI,POLY INT)
-> no appropriate / found in PositiveInteger
-> no appropriate / found in Polynomial Integer
-> no appropriate / found in Integer
-> no appropriate / found in PositiveInteger
-> no appropriate / found in Polynomial Integer
-> no appropriate / found in Integer

```

Modemaps from Associated Packages

no modemaps

So now it turns to the general modemaps:

Remaining General Modemaps

```

[1] (D,D1) -> D from D if D has VSPACE D1 and D1 has FIELD
[2] (D,D1) -> D from D
    if D has RMACAT(D2,D3,D1,D4,D5) and D1 has RING and D4 has
    DIRPCAT(D3,D1) and D5 has DIRPCAT(D2,D1) and D1 has FIELD
[3] (D1,D1) -> D from D if D has QFCAT D1 and D1 has INTDOM

```

```

[4] (D,D1) -> D from D
      if D has MATCAT(D1,D2,D3) and D1 has RING and D2 has FLAGG
      D1 and D3 has FLAGG D1 and D1 has FIELD
[5] (D,D1) -> D from D
      if D has LIECAT D1 and D1 has COMRING and D1 has FIELD
[6] (D,D) -> D from D if D has GROUP
[7] (SparseMultivariatePolynomial(D2,Kernel D),
      SparseMultivariatePolynomial(D2,Kernel D)) -> D
      from D if D2 has INTDOM and D2 has ORDSET and D has FS D2

[8] (Float,Integer) -> Float from Float
[9] (D,D) -> D from D if D has FIELD
[10] (D,D) -> D from D
      if D = EQ D1 and D1 has FIELD and D1 has TYPE or D = EQ D1
      and D1 has GROUP and D1 has TYPE
[11] (DoubleFloat,Integer) -> DoubleFloat from DoubleFloat
[12] (D,D1) -> D from D
      if D has AMR(D1,D2) and D1 has RING and D2 has OAMON and D1
      has FIELD

```

it eventually promotes PI to FRAC(POLY(INT)) and POLY(INT) to FRAC(POLY(INT)) and finds the match:

```
(FRAC(POLY(INT))) / (FRAC(POLY(INT)))
```

We can ask the interpreter to show us this operation (again, the output is truncated for brevity):

```

)show FRAC(POLY(INT))
Fraction Polynomial Integer is a domain constructor.
Abbreviation for Fraction is FRAC
This constructor is exposed in this frame.
Issue )edit src/algebra/FRAC.spad to see algebra source code for FRAC

```

```
----- Operations -----
```

```

???: (Fraction Integer,%) -> %      ???: (Integer,%) -> %
???: (PositiveInteger,%) -> %      ???: (%,Fraction Integer) -> %
???: (%,%) -> %                    ????: (%,Integer) -> %
????: (%,PositiveInteger) -> %     ?+?: (%,%) -> %
?-?: (%,%) -> %                    -?: % -> %
?/? : (%,%) -> %                    ?<? : (%,%) -> Boolean

```

```

[1] signature: (FRAC POLY INT,FRAC POLY INT) -> FRAC POLY INT
      implemented: slot $$$ from FRAC POLY INT

```

At this point the interpreter has succeeded in finding a type for the expression and eventually returns the result badged with the appropriate type:

```

      1
(1)  -----
      a x + b

```

```
Type: Fraction Polynomial Integer
```

A simple integral

Now we will show an integration with successive levels of expansion of explanation. We will use the expression above:

(1) `-> f:=1/(a*x+b)`

$$(1) \quad \frac{1}{a x + b}$$

Type: Fraction Polynomial Integer

(2) `-> integrate(f,x)`

$$(2) \quad \frac{\log(a x + b)}{a}$$

Type: Union(Expression Integer,...)

A simple integral, expansion 1 interpreter

(2) `-> integrate(f,x)`

Here we assume the previous discussion of modemap handling for the expression `f` and we only look at the modemap handling for the `integrate` function. We are looking for a modemap of the form:

```
integrate(FRAC(POLY(INT)),VARIABLE x)
```

So first we look in the domains of the arguments, that is, in Fraction Polynomial Integer, and Variable. Neither one succeeds:

Function Selection for `integrate`

```
Arguments: (FRAC POLY INT,VARIABLE x)
```

```
-> no appropriate integrate found in Fraction Polynomial Integer
```

```
-> no appropriate integrate found in Variable x
```

```
-> no appropriate integrate found in Fraction Polynomial Integer
```

```
-> no appropriate integrate found in Variable x
```

Modemaps from Associated Packages

```
no modemaps
```

Next we look at the general modemaps to find one that might work:

Remaining General Modemaps

```
[1] (D,D1) -> D from D
```

```
if D1 = SYMBOL and D has UTSCAT D2 and D2 has RING and D2
has ACFS INT and D2 has PRIMCAT and D2 has TRANFUN and D2
has ALGEBRA FRAC INT or D1 = SYMBOL and D has UTSCAT D2 and
D2 has RING and D2 has variables: D2 -> List D1 and D2 has
integrate: (D2,D1) -> D2 and D2 has ALGEBRA FRAC INT
```

```
[2] (D,D1) -> D from D
```

```
if D1 = SYMBOL and D has UPXSCAT D2 and D2 has RING and D2
has ACFS INT and D2 has PRIMCAT and D2 has TRANFUN and D2
```

```

has ALGEBRA FRAC INT or D1 = SYMBOL and D has UPXSCAT D2
and D2 has RING and D2 has variables: D2 -> List D1 and D2
has integrate: (D2,D1) -> D2 and D2 has ALGEBRA FRAC INT

```

- ```

[3] (D,D1) -> D from D
 if D1 = SYMBOL and D has ULSCAT D2 and D2 has RING and D2
 has ACFS INT and D2 has PRIMCAT and D2 has TRANFUN and D2
 has ALGEBRA FRAC INT or D1 = SYMBOL and D has ULSCAT D2 and
 D2 has RING and D2 has variables: D2 -> List D1 and D2 has
 integrate: (D2,D1) -> D2 and D2 has ALGEBRA FRAC INT
[4] (Polynomial D2,Symbol) -> Polynomial D2 from Polynomial D2
 if D2 has ALGEBRA FRAC INT and D2 has RING
[5] (D,D1) -> D from D
 if D has MTSCAT(D2,D1) and D2 has RING and D1 has ORDSET
 and D2 has ALGEBRA FRAC INT
[6] (Fraction Polynomial D4,Symbol) -> Union(Expression D4,List
 Expression D4)
 from IntegrationResultRFToFunction D4
 if D4 has CHARZ and D4 has Join(GcdDomain,RetractableTo
 Integer,OrderedSet,LinearlyExplicitRingOver Integer)
[7] (Expression Float,List Segment OrderedCompletion Float) ->
 Result
 from AnnaNumericalIntegrationPackage
[8] (Expression Float,Segment OrderedCompletion Float) -> Result
 from AnnaNumericalIntegrationPackage
[9] (GeneralUnivariatePowerSeries(D2,D3,D4),Variable D3) ->
 GeneralUnivariatePowerSeries(D2,D3,D4)
 from GeneralUnivariatePowerSeries(D2,D3,D4)
 if D3: SYMBOL and D2 has ALGEBRA FRAC INT and D2 has RING
 and D4: D2
[10] (D2,Symbol) -> Union(D2,List D2) from FunctionSpaceIntegration(
 D4,D2)
 if D4 has Join(EuclideanDomain,OrderedSet,
 CharacteristicZero,RetractableTo Integer,
 LinearlyExplicitRingOver Integer) and D2 has Join(
 TranscendentalFunctionCategory,PrimitiveFunctionCategory,
 AlgebraicallyClosedFunctionSpace D4)
[11] (Fraction Polynomial D4,SegmentBinding OrderedCompletion
 Fraction Polynomial D4) -> Union(f1: OrderedCompletion Expression
 D4,f2: List OrderedCompletion Expression D4,fail: failed,
 pole: potentialPole)
 from RationalFunctionDefiniteIntegration D4
 if D4 has Join(EuclideanDomain,OrderedSet,
 CharacteristicZero,RetractableTo Integer,
 LinearlyExplicitRingOver Integer)
[12] (Fraction Polynomial D4,SegmentBinding OrderedCompletion
 Expression D4) -> Union(f1: OrderedCompletion Expression D4,f2:
 List OrderedCompletion Expression D4,fail: failed,pole:
 potentialPole)
 from RationalFunctionDefiniteIntegration D4
 if D4 has Join(EuclideanDomain,OrderedSet,
 CharacteristicZero,RetractableTo Integer,
 LinearlyExplicitRingOver Integer)
[13] (D2,SegmentBinding OrderedCompletion D2) -> Union(f1:

```

```

OrderedCompletion D2,f2: List OrderedCompletion D2,fail: failed,
pole: potentialPole)
from ElementaryFunctionDefiniteIntegration(D4,D2)
if D2 has Join(TranscendentalFunctionCategory,
PrimitiveFunctionCategory,AlgebraicallyClosedFunctionSpace
D4) and D4 has Join(EuclideanDomain,OrderedSet,
CharacteristicZero,RetractableTo Integer,
LinearlyExplicitRingOver Integer)

```

Modemap [6] wins because we can construct the first argument by matching

```
Fraction Polynomial Integer
```

to

```
Fraction Polynomial D4
```

so we can infer that  $D4 == \text{Integer}$

```

[6] (Fraction Polynomial D4,Symbol) -> Union(Expression D4,List
Expression D4)
from IntegrationResultRFToFunction D4
if D4 has CHARZ and D4 has Join(GcdDomain,RetractableTo
Integer,OrderedSet,LinearlyExplicitRingOver Integer)

```

Given that match we have two requirements on Integer, both of which we can check with the interpreter:

INT has CHARZ

```
(3) true
```

```
Type: Boolean
```

```
(4) -> INT has Join(GcdDomain,RetractableTo Integer,OrderedSet,
LinearlyExplicitRingOver Integer)
```

```
(4) true
```

```
Type: Boolean
```

So we have a match

```

[1] signature: (FRAC POLY INT,SYMBOL) -> Union(EXPR INT,LIST EXPR INT)
implemented: slot (Union (Expression (Integer))
(List (Expression (Integer))))
(Fraction (Polynomial (Integer)))(Symbol)
from IRRF2F INT
[2] signature: (EXPR INT,SYMBOL) -> Union(EXPR INT,LIST EXPR INT)
implemented: slot (Union (Expression (Integer))
(List (Expression (Integer))))
(Expression (Integer))(Symbol)
from FSINT(INT,EXPR INT)

```

Now we invoke

```

integrate(FRAC(POLY(INT)),SYMBOL) -> Union(EXPR INT,LIST EXPR INT)
from IRRF2F(INT)

```

```
integrate(1/(a*x+b),x)
```

can print the result:

$$(2) \quad \frac{\log(a x + b)}{a}$$

```
Type: Union(Expression Integer,...)
```

## A simple integral, expansion 2 integrate

Now that we know how the interpreter has matched the input and called the function we need to follow the first level call into the function.

Axiom provides a trace tool that will allow us to walk into the function invocation and watch what happens. We will follow this same invocation path many times, each time we will descend another layer, repeating the information as we do.

For now, we look at the domain IRRF2F from irexand.spad. The categorical definition of this domain reads (we remove parts of the definition for brevity):

```
IntegrationResultRFToFunction(R): Exports == Implementation where
R: Join(GcdDomain, RetractableTo Integer, OrderedSet,
 LinearlyExplicitRingOver Integer)
```

```
RF ==> Fraction Polynomial R
F ==> Expression R
IR ==> IntegrationResult RF
OF ==> OutputForm
```

```
Exports ==> with
expand : IR -> List F
 ++ expand(i) returns the list of possible real functions
 ++ corresponding to i.
if R has CharacteristicZero then
 integrate : (RF, Symbol) -> Union(F, List F)
 ++ integrate(f, x) returns the integral of \spad{f(x)dx}
 ++ where x is viewed as a real variable..
```

```
Implementation ==> add
import IntegrationTools(R, F)
import TrigonometricManipulations(R, F)
import IntegrationResultToFunction(R, F)
```

```
toEF: IR -> IntegrationResult F
```

```
toEF i == map(#1::F, i)$IntegrationResultFunctions2(RF, F)
expand i == expand toEF i
complexExpand i == complexExpand toEF i
```

```
if R has CharacteristicZero then
```

```

import RationalFunctionIntegration(R)

if R has imaginary: () -> R then
 integrate(f, x) == complexIntegrate(f, x)
else
 integrate(f, x) ==
 l := [mkPrim(real g, x) for g in expand internalIntegrate(f, x)]
 empty? rest l => first l
 l

```

@

We can see that this domain constructor takes one argument which, in this case, is Integer. We've already determined that Integer has the required Joins:

```
(4) -> INT has Join(GcdDomain,RetractableTo Integer,OrderedSet,
 LinearlyExplicitRingOver Integer)
```

```
(4) true
```

Type: Boolean

and we can see that:

```
(5) -> INT has CharacteristicZero
```

```
(5) true
```

Type: Boolean

so we can match the signature of integrate:

```

integrate(Fraction Polynomial Integer, Symbol) ->
 Union(Expression Integer, List Expression Integer)

```

We can trace this domain and ask to see the output in math form:

```
(6) ->)trace IRRF2F)math
```

```

Packages traced:
 IntegrationResultRFToFunction Integer
Parameterized constructors traced:
 IRRF2F

```

and now, when we do the integration, we see the output of the trace:

```

integrate(1/(a*x+b),x)
1<enter IntegrationResultRFToFunction.integrate,32 :
 1
 arg1= -----
 a x + b
 arg2= x
1<enter IntegrationResultRFToFunction.expand,18 :
 1 a x + b
 arg1= - log(-----)
 a a
1>exit IntegrationResultRFToFunction.expand,18 :
 a x + b
 log(-----)
 a
 [-----]

```

```

 a
1>exit IntegrationResultRFToFunction.integrate,32 :
log(a x + b)

 a

 log(a x + b)
(6) -----
 a

 Type: Union(Expression Integer,...)

```

From this we learn that the arguments to integrate are exactly the arguments we supplied and we know the exact types of the arguments because they have to match the signature of the function:

```

1<enter IntegrationResultRFToFunction.integrate,32 :
integrate(, Symbol) ->
 1
arg1= ----- <== Fraction Polynomial Integer
 a x + b
arg2= x <== Symbol

```

and returns the result

```

1>exit IntegrationResultRFToFunction.integrate,32 :
log(a x + b)
----- <== Union(Expression Integer, List Expression Integer)
 a

```

## A simple integral, expansion 2 internalIntegrate

If we look at the function definition for integrate:

```

integrate(f, x) ==
 l := [mkPrim(real g, x) for g in expand internalIntegrate(f, x)]
 empty? rest l => first l
 l

```

we can see that there is a call to the function

```
internalIntegrate(f, x)
```

and we can compute the types of the arguments since they are exactly the types of the integrate function itself:

```
internalIntegrate(Fraction Polynomial Integer, Symbol)
```

and since the return value will be fed to the expand function we can look at the signature of expand:

```
expand: IntegrationResult Fraction Polynomial Integer ->
 List Expression Integer

```

and we can get the full signature for internalIntegrate:

```
internalIntegrate(Fraction Polynomial Integer, Symbol) ->
 IntegrationResult Fraction Polynomial Integer

```

This comes from the domain



```
RationalFunctionIntegration(F): Exports == Implementation where
 F: Join(IntegralDomain, RetractableTo Integer, CharacteristicZero)
```

where F is Integer.

```
SE ==> Symbol
P ==> Polynomial F
Q ==> Fraction P
UP ==> SparseUnivariatePolynomial Q
QF ==> Fraction UP
LGQ ==> List Record(coeff:Q, logand:Q)
UQ ==> Union(Record(ratpart:Q, coeff:Q), "failed")
ULQ ==> Union(Record(mainpart:Q, limitedlogs:LGQ), "failed")

Exports ==> with
 internalIntegrate: (Q, SE) -> IntegrationResult Q
 ++ internalIntegrate(f, x) returns g such that \spad{dg/dx = f}.
Implementation ==> add
 import RationalIntegration(Q, UP)
 import IntegrationResultFunctions2(QF, Q)
 import PolynomialCategoryQuotientFunctions(IndexedExponents SE,
 SE, F, P, Q)

 internalIntegrate(f, x) ==
 map(multivariate(#1, x), integrate univariate(f, x))
```

If we look the signature for internalIntegrate and expand it we see:

```
internalIntegrate: (Q, SE) -> IntegrationResult Q

internalIntegrate: (Fraction Polynomial Integer, Symbol) ->
 IntegrationResult Fraction Polynomial Integer
```

which is exactly what we need. When we look at the function we see:

```
internalIntegrate(f, x) ==
 map(multivariate(#1, x), integrate univariate(f, x))
```

We can watch the function call by tracing INTRF:

```
(7) ->)trace INTRF)math
```

```
Packages traced:
 IntegrationResultRFTToFunction Integer,
 RationalFunctionIntegration Integer
Parameterized constructors traced:
 IRRF2F, INTRF
```

and we see:

```
(7) -> integrate(1/(a*x+b),x)
1<enter IntegrationResultRFTToFunction.integrate,32 :
 1
arg1= -----
 a x + b
arg2= x
1<enter RationalFunctionIntegration.internalIntegrate,25 :
 1
arg1= -----
```

```

 a x + b
arg2= x
1>exit RationalFunctionIntegration.internalIntegrate,25 :
1 a x + b
- log(-----)
a a
1<enter IntegrationResultRFToFunction.expand,18 :
1 a x + b
arg1= - log(-----)
a a
1>exit IntegrationResultRFToFunction.expand,18 :
 a x + b
log(-----)
 a
[-----]
 a
1>exit IntegrationResultRFToFunction.integrate,32 :
log(a x + b)

 a

 log(a x + b)
(7) -----
 a

```

Type: Union(Expression Integer,...)

Now we see that `internalIntegrate` was called with the arguments

```

1<enter RationalFunctionIntegration.internalIntegrate,25 :
1
arg1= ----- <== Fraction Polynomial Integer
 a x + b
arg2= x <== Symbol

```

and returned the values:

```

1>exit RationalFunctionIntegration.internalIntegrate,25 :
1 a x + b
- log(-----) <== IntegrationResult Fraction Polynomial Integer
a a

```

## A simple integral, expansion 3 univariate

But the `internalIntegrate` function does its work by calling yet other functions, the deepest of which is univariate:

```

internalIntegrate(f, x) ==
 map(multivariate(#1, x), integrate univariate(f, x))

```

Since `univariate` uses the arguments to the `internalIntegrate` function which has the signature:

```

internalIntegrate: (Fraction Polynomial Integer, Symbol) ->

```

we can determine that we need a univariate function with the signature:

```

univariate: (Fraction Polynomial Integer, Symbol) ->

```

This function is found in `PolynomialCategoryQuotientFunctions`, `POLYCATQ` which has

the form:

```
PolynomialCategoryQuotientFunctions(E, V, R, P, F):
Exports == Implementation where
E: OrderedAbelianMonoidSup
V: OrderedSet
R: Ring
P: PolynomialCategory(R, E, V)
F: Field with
 coerce: P -> %
 numer : % -> P
 denom : % -> P

UP ==> SparseUnivariatePolynomial F
RF ==> Fraction UP

Exports ==> with
variables : F -> List V
 ++ variables(f) returns the list of variables appearing
 ++ in the numerator or the denominator of f.
mainVariable: F -> Union(V, "failed")
 ++ mainVariable(f) returns the highest variable appearing
 ++ in the numerator or the denominator of f, "failed" if
 ++ f has no variables.
univariate : (F, V) -> RF
 ++ univariate(f, v) returns f viewed as a univariate
 ++ rational function in v.
Implementation ==> add
P2UP: (P, V) -> UP

univariate(f, x) == P2UP(numer f, x) / P2UP(denom f, x)

P2UP(p, x) ==
 map(#1::F,
 univariate(p, x))$SparseUnivariatePolynomialFunctions2(P, F)
```

So we are calling the function:

```
univariate: (Fraction Polynomial Integer, Symbol) ->
Fraction SparseUnivariatePolynomial Field with
coerce: PolynomialCategory(Ring, OrderedAbelianMonoidSup, OrderedSet) -> %
numer: % -> PolynomialCategory(Ring, OrderedAbelianMonoidSup, OrderedSet)
denom: % -> PolynomialCategory(Ring, OrderedAbelianMonoidSup, OrderedSet)
```

which we can see by tracing that domain:

```
(8) ->)trace POLYCATQ)math
```

Packages traced:

```
IntegrationResultRFToFunction Integer,
RationalFunctionIntegration Integer,
PolynomialCategoryQuotientFunctions(IndexedExponents
Kernel Expression Integer,Kernel Expression Integer,
Integer,SparseMultivariatePolynomial(Integer,Kernel
Expression Integer),Expression Integer),
PolynomialCategoryQuotientFunctions(IndexedExponents
Symbol,Symbol,Integer,Polynomial Integer,Fraction
```

```

 Polynomial Integer)
Parameterized constructors traced:
 IRRF2F, INTRF, POLYCATQ

```

which gives the input:

```

1>enter PolynomialCategoryQuotientFunctions.univariate,16 :
 1
arg1= ----- <== Fraction Polynomial Integer
 a x + b
arg2= x <== Symbol

```

and the output

```

1>exit PolynomialCategoryQuotientFunctions.univariate,16 :
 1
 -
 a
----- <== Fraction SparseUnivariatePolynomial Field with
 b coerce: P -> %
? + - numer: % -> P
 a denom: % -> P

```

It should be clear that univariate divided the numerator and denominator by the leading coefficient of the polynomial in the denominator. It also replaced “x” with the variable “?”.

## A simple integral, expansion 4 integrate

When univariate returns, the results are fed to another integrate, this time from Rational-Integration (INTRAT). This domain looks like:

```

RationalIntegration(F, UP): Exports == Implementation where
 F : Join(Field, CharacteristicZero, RetractableTo Integer)
 UP: UnivariatePolynomialCategory F

RF ==> Fraction UP
IR ==> IntegrationResult RF
LLG ==> List Record(coeff:RF, logand:RF)
URF ==> Union(Record(ratpart:RF, coeff:RF), "failed")
U ==> Union(Record(mainpart:RF, limitedlogs:LLG), "failed")
OF ==> OutputForm

Exports ==> with
 integrate : RF -> IR
 ++ integrate(f) returns g such that \spad{g' = f}.

Implementation ==> add
 import TranscendentalIntegration(F, UP)

 integrate f ==
 rec := monomialIntegrate(f, differentiate)
 integrate(rec.polypart)::RF::IR + rec.ir

```

This domain was constructed and “brought into scope” in RationalFunctionIntegration(F) with the statement

```
import RationalIntegration(Fraction Polynomial Integer,
 SparseUnivariatePolynomial Fraction Polynomial Integer)
```

and the function has the signature

```
integrate:
 Fraction SparseUnivariatePolynomial Fraction Polynomial Integer ->
 IntegrationResult Fraction
 Fraction Polynomial Integer

1<enter RationalIntegration.integrate,32 :
 1
 -
 a
arg1= ----- <== Fraction SparseUnivariatePolynomial
 b Fraction Polynomial Integer
? + -
 a
1>exit RationalIntegration.integrate,32 :
 1 b
- log(? + -) <== IntegrationResult Fraction SparseUnivariatePolynomial
a a Fraction Polynomial Integer
```

## A simple integral, expansion 5 monomialIntegrate

The integrate function is defined as:

```
integrate f ==
 print(outputForm("tpdhere INTRAT 1")@OF)$OF
 rec := monomialIntegrate(f, differentiate)
 integrate(rec.polypart)::RF::IR + rec.ir
```

Notice that while “f” is an argument to integrate, the “differentiate” function is a free variable. The Axiom compiler will look at all of the symbols “in scope” to find its meaning. This code does an import:

```
import TranscendentalIntegration(Fraction Polynomial Integer,
 SparseUnivariatePolynomial Fraction Polynomial Integer)
```

which exports monomialIntegrate

TranscendentalIntegration(F, UP): Exports == Implementation where

```
F : Field
UP : UnivariatePolynomialCategory F

RF ==> Fraction UP
FF ==> Record(ratpart:F, coeff:F)
UF ==> Union(FF, "failed")
IR ==> IntegrationResult RF
REC ==> Record(ir:IR, specpart:RF, polypart:UP)
```

Exports ==> with

```
monomialIntegrate : (RF, UP -> UP) -> REC
++ monomialIntegrate(f, ') returns \spad{[ir, s, p]} such that
++ \spad{f = ir' + s + p} and all the squarefree factors of the
++ denominator of s are special w.r.t the derivation '.
```

```

Implementation ==> add
import SubResultantPackage(UP, UP2)
import MonomialExtensionTools(F, UP)
import TranscendentalHermiteIntegration(F, UP)
import CommuteUnivariatePolynomialCategory(F, UP, UP2)

monomialIntegrate(f, derivation) ==
 zero? f => [0, 0, 0]
 r := HermiteIntegrate(f, derivation)
 zero?(inum := numer(r.logpart)) =>
 [r.answer::IR, r.specpart, r.polypart]
 iden := denom(r.logpart)
 x := monomial(1, 1)$UP
 resultvec := subresultantVector(UP2UP2 inum -
 (x::UP2) * UP2UP2 derivation iden, UP2UP2 iden)
 respoly := primitivePart leadingCoefficient resultvec 0
 rec := splitSquarefree(respoly, kappa(#1, derivation))
 logs:List(LOG) := [
 [1, UP2UPR(term.factor),
 UP22UPR swap primitivePart(resultvec(term.exponent), term.factor)]
 for term in factors(rec.special)]
 dlog :=
 ((derivation x) = 1) => r.logpart
 differentiate(mkAnswer(0, logs, empty()),
 differentiate(#1, derivation))
 (u := retractIfCan(p := r.logpart - dlog)@Union(UP, "failed")) case UP =>
 [mkAnswer(r.answer, logs, empty), r.specpart, r.polypart + u::UP]
 [mkAnswer(r.answer, logs, [[p, dummy]]), r.specpart, r.polypart]

```

which expands into the type signature:

```

monomialIntegrate:
(Fraction SparseUnivariatePolynomial Fraction Polynomial Integer,
 SparseUnivariatePolynomial Fraction Polynomial Integer ->
 SparseUnivariatePolynomial Fraction Polynomial Integer) ->
Record(ir: IntegrationResult Fraction
 SparseUnivariatePolynomial Fraction Polynomial Integer,
 specpart: Fraction
 SparseUnivariatePolynomial Fraction Polynomial Integer,
 polypart: SparseUnivariatePolynomial Fraction Polynomial Integer)
++ monomialIntegrate(f, ') returns \spad{[ir, s, p]} such that
++ \spad{f = ir' + s + p} and all the squarefree factors of the
++ denominator of s are special w.r.t the derivation '.

```

we can watch this happen:

```
)trace INTTR)math
```

```

Function traced: UnivariatePolynomialCategory
Packages traced:
 IntegrationResultRFToFunction Integer,
 RationalFunctionIntegration Integer,
 RationalIntegration(Fraction Polynomial Integer,
 SparseUnivariatePolynomial Fraction Polynomial

```

```

Integer), PolynomialCategoryQuotientFunctions(
IndexedExponents Kernel Expression Integer,Kernel
Expression Integer,Integer,
SparseMultivariatePolynomial(Integer,Kernel
Expression Integer),Expression Integer),
PolynomialCategoryQuotientFunctions(IndexedExponents
Symbol,Symbol,Integer,Polynomial Integer,Fraction
Polynomial Integer), TranscendentalIntegration(
Fraction Polynomial Integer,
SparseUnivariatePolynomial Fraction Polynomial
Integer)

```

Parameterized constructors traced:

```
IRRF2F, INTRF, INTRAT, POLYCATQ, INTTR
```

and we can watch the monomialIntegrate function call

```

(34) -> integrate(1/(a*x+b),x)
1<enter IntegrationResultRFToFunction.integrate,32 :
 1
arg1= -----
 a x + b
arg2= x
 "tpdhere IRRF2F 1"
1<enter RationalFunctionIntegration.internalIntegrate,25 :
 1
arg1= -----
 a x + b
arg2= x
1<enter PolynomialCategoryQuotientFunctions.univariate,16 :
 1
arg1= -----
 a x + b
arg2= x
1>exit PolynomialCategoryQuotientFunctions.univariate,16 :
 1
 -
 a

 b
? + -
 a
1<enter RationalIntegration.integrate,32 :
 1
 -
 a
arg1= ----- <== Fraction SparseUnivariatePolynomial
 b Fraction Polynomial Integer
? + -
 a
1<enter TranscendentalIntegration.monomialIntegrate,81 :
 1
 -
 a
arg1= ----- <== Fraction SparseUnivariatePolynomial
 b Fraction Polynomial Integer

```

```

 ? + -
 a
 arg2= theMap(UPOLYC-;differentiate;2S;37,873)
1>exit TranscendentalIntegration.monomialIntegrate,81 :
 1 b
 [ir= - log(? + -),specpart= 0,polypart= 0]
 a a
1>exit RationalIntegration.integrate,32 :
 1 b
 - log(? + -)
 a a
1>exit RationalFunctionIntegration.internalIntegrate,25 :
 1 a x + b
 - log(-----)
 a a
1>exit IntegrationResultRFToFunction.integrate,32 :
log(a x + b)

 a

 log(a x + b)
(34) -----
 a
Type: Union(Expression Integer,...)
(35) ->

```

## A simple integral, expansion 6 HermiteIntegrate

Since “f” is not zero we invoke HermiteIntegrate from the domain TranscendentalHermiteIntegration which looks like:

TranscendentalHermiteIntegration(F, UP): Exports == Implementation where

F : Field

UP : UnivariatePolynomialCategory F

N ==> NonNegativeInteger

RF ==> Fraction UP

REC ==> Record(answer:RF, lognum:UP, logden:UP)

HER ==> Record(answer:RF, logpart:RF, specpart:RF, polypart:UP)

Exports ==> with

HermiteIntegrate: (RF, UP -> UP) -> HER

++ HermiteIntegrate(f, D) returns \spad{[g, h, s, p]}

++ such that \spad{f = Dg + h + s + p},

++ h has a squarefree denominator normal w.r.t. D,

++ and all the squarefree factors of the denominator of s are

++ special w.r.t. D. Furthermore, h and s have no polynomial parts.

++ D is the derivation to use on \spadtype{UP}.

Implementation ==> add

import MonomialExtensionTools(F, UP)

HermiteIntegrate(f, derivation) ==

rec := decompose(f, derivation)



```

hi := normalHermiteIntegrate(rec.normal, derivation)
qr := divide(hi.lognum, hi.logden)
[hi.answer, qr.remainder / hi.logden, rec.special, qr.quotient + rec.poly]

```

The function has the same input signature as `monomialIntegrate` but a different return signature.

```

HermiteIntegrate:
(Fraction SparseUnivariatePolynomial Fraction Polynomial Integer,
 SparseUnivariatePolynomial Fraction Polynomial Integer ->
 SparseUnivariatePolynomial Fraction Polynomial Integer) ->
Record(answer:Fraction SparseUnivariatePolynomial
 Fraction Polynomial Integer,
 logpart:Fraction SparseUnivariatePolynomial
 Fraction Polynomial Integer,
 specpart:Fraction SparseUnivariatePolynomial
 Fraction Polynomial Integer,
 polypart:Fraction SparseUnivariatePolynomial Fraction Polynomial Integer)

```

so we trace this domain

```
(37) ->)trace INTHERTR)math
```

```

Function traced: UnivariatePolynomialCategory
Packages traced:
 IntegrationResultRFToFunction Integer,
 RationalFunctionIntegration Integer,
 RationalIntegration(Fraction Polynomial Integer,
 SparseUnivariatePolynomial Fraction Polynomial
 Integer), PolynomialCategoryQuotientFunctions(
 IndexedExponents Kernel Expression Integer,Kernel
 Expression Integer,Integer,
 SparseMultivariatePolynomial(Integer,Kernel
 Expression Integer),Expression Integer),
 PolynomialCategoryQuotientFunctions(IndexedExponents
 Symbol,Symbol,Integer,Polynomial Integer,Fraction
 Polynomial Integer), TranscendentalIntegration(
 Fraction Polynomial Integer,
 SparseUnivariatePolynomial Fraction Polynomial
 Integer), TranscendentalHermiteIntegration(Fraction
 Polynomial Integer,SparseUnivariatePolynomial
 Fraction Polynomial Integer)
Parameterized constructors traced:
 IRRF2F, INTRF, INTRAT, POLYCATQ, INTTR, INTHERTR

```

and now we see

```

(38) -> integrate(1/(a*x+b),x)
1<enter IntegrationResultRFToFunction.integrate,32 :
 1
arg1= -----
 a x + b
arg2= x
 "tpdhere IRRF2F 1"
1<enter RationalFunctionIntegration.internalIntegrate,25 :
 1
arg1= -----

```

```

 a x + b
arg2= x
1<enter RationalIntegration.integrate,32 :
 1
 -
 a
arg1= -----
 b
 ? + -
 a
"tpdhere INTRAT 1"
1<enter TranscendentalIntegration.monomialIntegrate,81 :
 1
 -
 a
arg1= -----
 b
 ? + -
 a
arg2= theMap(UPOLYC-;differentiate;2S;37,873)
1<enter TranscendentalHermiteIntegration.HermiteIntegrate,18 :
 1
 -
 a
arg1= -----
 b
 ? + -
 a
arg2= theMap(UPOLYC-;differentiate;2S;37,873)
1>exit TranscendentalHermiteIntegration.HermiteIntegrate,18 :
 1
 -
 a
[answer= 0,logpart= -----,specpart= 0,polypart= 0]
 b
 ? + -
 a
1>exit TranscendentalIntegration.monomialIntegrate,81 :
 1 b
[ir= - log(? + -),specpart= 0,polypart= 0]
 a a
"tpdhere UPOLYC 1"
1>exit RationalIntegration.integrate,32 :
 1 b
- log(? + -)
 a a
1>exit RationalFunctionIntegration.internalIntegrate,25 :
 1 a x + b
- log(-----)
 a a
1<enter IntegrationResultRFTtoFunction.expand,18 :
 1 a x + b
arg1= - log(-----)
 a a

```

```

1>exit IntegrationResultRFToFunction.expand,18 :
 a x + b
 log(-----)
 a
 [-----]
 a
1>exit IntegrationResultRFToFunction.integrate,32 :
log(a x + b)

 a

 log(a x + b)
(38) -----
 a

 Type: Union(Expression Integer,...)

```

so HermiteIntegrate did nothing to the input. Next we call normalHermiteIntegrate which is a local function

## 0.5 Tools

### svn

SVN is a source control system on all platforms. Axiom 'silver' is maintained in an SVN archive on sourceforge. This can be pulled from:

```
svn co https://axiom.svn.sf.net/svnroot/axiom/trunk/axiom axiom
```

### git

Git is a unix-based source code control system. Axiom 'silver' is maintained in a git archive. This can be pulled from:

```
git-clone ssh://git@axiom-developer.org/home/git/silver
```

the password for the userid git is linux.

### CVS

This assumes that you have set up ssh on the Savannah site. CVS does not use a password. You have to log onto the Savannah site and set up a public key. This requires you to:

- set up a local public key: `ssh-keygen -b 1024 -t rsa1`
- open a browser
- navigate to the savannah page that has your personal keys
- open `.ssh/identity.pub`
- cut `.ssh/identity.pub`
- paste it into your personal key list on savannah
- go have a beer (the page takes an hour or two to update)

Once you have a working key you can do the cvs login. If it prompts you for a password then the key is not working. If it prompts you to “Enter the passphrase for RSA key” then cvs login will work.

I maintain a directory where I work (call this WORK)

```
/home/axiomgnu/new
```

and a directory for CVS (call this GOLD)

```
/axiom
```

When I want to export a set of changes I do the following steps:

0) MAKE SURE THE /.ssh/config FILE IS CORRECT:

(you should only need to do this once.  
you need to change the User= field)

```
Host *.gnu.org
 Protocol=1
 Compression=yes
 CompressionLevel=3
 User=axiom
 StrictHostKeyChecking=no
 PreferredAuthentications=publickey,password
 NumberOfPasswordPrompts=2
```

1) MAKE SURE THE SHELL VARIABLES ARE OK:

(normally set in .bashrc)

```
export CVS_RSH=ssh
export CVSROOT=:pserver:axiom@subversions.gnu.org:/cvsroot/axiom
      ~~~~~
      change this to your id
```

2) MAKE SURE YOU'RE LOGGED IN:

(I keep a session open all the time but it doesn't seem to care  
if you login again. i'm not sure what login does, actually)

```
cvs login
```

3) GET A FRESH COPY FOR THE FIRST TIME OR AT ANY TIME:

(you only need to do this the first time but you can erase  
your whole axiom subtree and refresh it again doing this.

note that i work as root so i can update /. Most rational  
people are smarter than me and work as a regular user so  
you have to change the instructions for cd. But you knew that)

```
cd /
cvs co axiom
```

4) MAKE SURE THAT GOLD, MY LOCAL CVS COPY, IS UP TO DATE:

(I maintain an exact copy of the CVS repository and only make  
changes to it when i want to export the changes. that way I  
won't export my working tree by accident. my working tree is  
normally badly broken.

The update command makes sure that you have all of the changes other people might have made and checked in. you have to merge your changes so you don't step on other people's work. So be sure to run update BEFORE you copy files to GOLD)

```
cd /axiom
cvs update
```

#### 5) COPY CHANGED FILES FROM WORK TO THE GOLD TREE:

(This is an example for updating the \*.daase files.  
You basically are changing your GOLD tree to reflect the way you want CVS to look once you check in all of the files.)

```
cd /home/axiomgnu/new
cp src/share/algebra/*.daase /axiom/src/share/algebra
```

#### 6) IF A FILE IS NEW (e.g. src/interp/foo.lisp.pamphlet) THEN:

(If you create a file you need to "put it under CVS control" CVS only cares about files you explicitly add or delete. If you make a new file and copy it to GOLD you need to do this.

Don't do the "cvs add" in your WORK directory. The cvs add command updates the files in the CVS directory and you won't have them in your WORK directory.

Notice that you do the "cvs add" in the directory where the file was added (hence, the cd commands).

```
cd /axiom/src/interp
cvs add -m"some pithy comment" foo.lisp.pamphlet
cd /axiom
```

#### 7) IF A FILE IS DELETED (e.g. src/interp/foo.lisp.pamphlet) THEN:

(you have to delete the file from the GOLD directory BEFORE you do a "cvs remove". The "cvs remove" will update the files in the CVS directory

Notice that you do the "cvs remove" in the directory where the file was deleted (hence, the cd commands).

```
cd /axiom/src/interp
rm foo.lisp.pamphlet
cvs remove foo.lisp.pamphlet
cd /axiom
```

#### 8) IF A DIRECTORY IS NEW (e.g. foodir) THEN:

(this will put "foodir" under CVS control. It will also create foodir/CVS as a directory with a bunch of control files in the foodir/CVS directory. Don't mess with the control files.

(there are a bunch of special rules about directories. empty directories are not downloaded by update.)

(NOTE: THERE IS NO WAY TO DELETE A DIRECTORY)

```
cd /axiom/src
mkdir foodir
cvs add -m "pithy comment" foodir
cd /axiom
```

#### 9) EDIT CHANGELOG:

changelog is already under CVS control so it will get uploaded when you do the checkin.)

```
cd /axiom
emacs -nw changelog
(add a date, initials, and pithy comment, save it, and exit)
```

#### 10) CHECK IN THE CHANGES

(This will actually change the savannah CVS repository.

The "cvs ci" command will recurse thru all of the lower subdirectories and look for changed files. It will change the host versions of those files to agree with your copy. If somebody else has changed a file while you were busy developing code then the checkin MAY complain (if it can't merge the changes)

```
cd /axiom
cvs ci -m"pithy comment"
```

Congrats. You've now done your first change to the production image. Please be very careful as this is a world readable copy. We don't want to ship nonsense. Test everything. Even trivial changes before you upload.

## 0.6 Common Lisps

### GCL

Axiom was ported to run under AKCL which was a common lisp developed by Bill Schelter. He started with KCL (Kyoto Common Lisp) and, since he lived and worked in Austin, Texas, named his version AKCL (Austin-Kyoto Common Lisp). Bill worked under contract to the Scratchpad group at IBM Research. I was the primary developer for system internals so Bill and I worked closely together on a lot of issues. After Axiom was sold to NAG Bill continued to develop AKCL and it eventually became GCL (Gnu Common Lisp).

In order to port Axiom to run on GCL we need to do several things. First, we need to apply a few patches. These patches enlarge the default stack size, remove the startup banner, link with Axiom's socket library, and rename collectfn.

The issue with the stack size is probably bogus. At one point the system was running out of stack space but the problem was due to a recursive expansion of a macro and no amount of stack space would be sufficient. This patch remains at the moment but should probably be removed and tested.

The startup banner is an issue because we plan to run under various frontend programs like

Texmacs and the Magnus ZLC. We need to just output a single prompt.

Axiom has a socket library because at the time it was developed under AKCL there was no socket code in Lisp. There is still not a standard common lisp socket library but I believe all common lisps have a way to manipulate sockets. This code should be rewritten in lisp and `#+` for each common lisp.

The `collectfn` file is a major optimization under GCL. When `collectfn` is loaded and the lisp compiler is run then `collectfn` will output a `.fn` file. The second time the compiler is invoked the `.fn` file is consulted to determine the actual types of arguments used. Function calling is highly optimized using this type information so that fast function calling occurs. Axiom should be built one time to create the `int/*/*.fn` files. It should then be rebuilt using the cached `.fn` files. I will automate this process into the Makefiles in the future.

GCL implementation will have a major porting problem to brand new platforms. The compiler strategy is to output C code, compile it using GCC, and dynamically link the machine code to the running image. This requires deep knowledge of the symbol tables used by the native linker for each system. In general this is a hard problem that requires a lot of expertise. Bill Schelter and I spent a lot of time and effort making this work for each port. The magic knowledge is not written down anywhere and I no longer remember the details.

## CCL

When Axiom was sold to NAG it was ported to CCL (Codemist Common Lisp) which is not, strictly speaking, a common lisp implementation. It contains just enough common lisp to support Axiom and, as I'm a great believer in simple code, it only needed a small subset of a full common lisp.

CCL can be considered the best way to get Axiom running on a new architecture as the porting issues are minimal.

CCL is a byte-interpreter implementation and has both the positive and negative aspects of that design choice. The positive aspect is that porting the code to run on new architectures is very simple. Once the CCL byte-code interpreter is running then Axiom is running. The saved-system image is pure byte-codes and is completely system independent.

The negative aspects are that it is slow and the garbage collector appears broken. Compiling the Axiom library files on an file-by-file basis takes about 1 hour on GCL and about 12 hours on CCL. Compiling all of the Axiom library files in the same image (as opposed to starting a new image per file) still takes about 1 hour on GCL. It never finishes in CCL. Indeed it stops doing useful work after about the 40th file (out of several hundred).

When Axiom became open source I moved the system back to GCL because I could not understand how to build a CCL system. I plan to revisit this in the future and document the process so others can follow it as well as build Makefiles to automate it.

## CMU CL

CMU CL grew out of the Carnegie-Mellon University SPICE project. That project studied the issues involved in building an optimizing compiler for common lisp. Axiom, back when it was Scratchpad at IBM, ran on CMU CL. Indeed, a lot of the lisp-level optimizations are due to use of the CMU CL compiler and the disassemble function.

### **Franz Lisp**

Axiom, as Scratchpad, ran on Franz Lisp.

### **Lucid Common Lisp**

Axiom, as Scratchpad, ran on Lucid Common Lisp.

### **Symbolics Common Lisp**

Axiom, as Scratchpad, ran on Symbolics Common Lisp.

### **Golden Common Lisp**

Axiom, as Scratchpad, ran on Golden Common Lisp. This was a PC version of Common Lisp which appears to have died.

### **VM/LISP 370**

Axiom, as Scratchpad, ran on VM/Lisp 370. This was an IBM version of lisp and was not a common lisp. The .daase random access file format is an artifact of running on this lisp.

### **Maclisp**

Axiom, as Scratchpad, ran on Maclisp. This was an early MIT version of lisp and is not common lisp. Many of the funny function names that have slightly different semantics than their common lisp counterparts still exist in the system as macros due to this lisp.

## **0.7 Changing GCL versions**

Axiom lives on GNU Common Lisp. Axiom adds C code to the lisp image. Axiom caches versions to ensure that nothing breaks. Changing GCL versions has introduced subtle bugs at various times. The steps necessary to introduce a new version are

1. Add the latest GCL sources to Axiom
2. Update the patches to the new version
3. create diff -Naur patches to the gcl sources
4. update lsp/Makefile.pamphlet to apply the patch at build
5. add a new chunk to lsp/Makefile.pamphlet to build gcl-2.6.10
6. Change the GCLVERSION to point at the new sources
7. change the Makefile to match Makefile.pamphlet

We assume in the following that `$AXHOME` is the home directory and that Axiom lives in the silver subdirectory.



In more detail these steps are:

1. Add the latest GCL sources to Axiom
  - (a) Download the latest GCL from gnu.org  
For these instructions assume the file is  
`gcl-Version_2_6_10.tar.gz`
  - (b) move the tar file into /tmp  
We are going to make changes to the distribution via patches
  - (c) untar the file  
`tar -zxf gcl-Version_2_6_10.tar.gz`
  - (d) cd to the untarred directory  
`cd gcl-Version_2_6_10`
  - (e) rename the gcl directory  
Camm follows a convention that the top level directory in the tar file is called gcl.  
Since we maintain several past versions we need to rename this and re-tar it  
`mv gcl gcl-2.6.10`
  - (f) rename gcl to use our naming convention  
`tar -zcf gcl-2.6.10.tgz gcl-2.6.10`
  - (g) We move the original, renamed, retarred file to the zip directory  
`mv gcl-2.6.10.tgz $AXHOME/silver/zips`
  - (h) We have to make sure to include the new file in the git commit  
`cd $AXHOME/silver`
  - (i) Tell git we care about the file  
`git add zips/gcl-2.6.10.tgz`
2. Update the patches to the new version
  - (a) find the previous patches  
`ls $AXHOME/silver/zips/gcl-2.6.8pre7*patch`
  - (b) for each patch do ( Step 3 ; Step 4 )
3. create diff -Naur patches to the gcl sources
  - (a) assume we are looking at gcl-2.6.8pre7.h.linux.defs.patch  
The name tells us what file to patch. From the above we can see that when Axiom builds GCL it will  
`cd lsp/gcl-2.6.8pre7`  
because GCL is built by the lsp/Makefile.pamphlet. That Makefile will do a  
`cd h`  
`patch <gcl-2.6.8pre7.h.linux.defs.patch`  
which will apply the patch .... So we need to make a patch
  - (b) move to the subdirectory containing the file  
`cd /tmp/gcl-Version_2_6_10/gcl-2.6.10/h`
  - (c) edit the 'linux.defs' file to create the proper patch

(d) save the changed file as `linux.defs.tpd`

(e) in a shell, create a diff -Naur patch by

```
diff -Naur linux.defs linux.defs.tpd >gcl-2.6.10.h.linux.defs.patch
```

(f) move it to the zips directory

```
mv gcl-2.6.10.h.linux.defs.patch $AXHOME/silver/zips
cd $AXHOME/silver
git add zips/gcl-2.6.10.h.linux.defs.patch
```

4. update `lsp/Makefile.pamphlet` to apply the patch at build

(a) edit `lsp/Makefile.pamphlet`

(b) search for chunk `gcl-2.6.8pre7.h.linux.defs.patch`

(c) copy the chunk and name the new chunk `gcl-2.6.10.h.linux.defs.patch`

5. add a new chunk to `lsp/Makefile.pamphlet` to build `gcl-2.6.10`

(a) find the subsection “The GCL-2.6.8pre7 stanza”

(b) make a copy named “The GCL-2.6.10 stanza”

(c) add the new patches

(d) tell git we care

```
cd $AXHOME/silver
git add lsp/Makefile.pamphlet
```

6. Change the `GCLVERSION` to point at the new sources

(a) emacs `$AXHOME/silver/Makefile.pamphlet`

(b) search for `#GCLVERSION`, a Makefile comment line

(c) the last line is uncommented. Assume it reads `GCLVERSION=gcl-2.6.8pre7`  
`gcl-2.6.8pre7` was is the name of the current version we are replacing. We will use  
 this name in the next step

(d) put a `#` in front of the `GCLVERSION` variable to comment it out

We maintain the list of old, working patches. We also remember the names of the  
 prior `GCLVERSIONS` in case we have to back up

(e) Add a new line reading:

```
GCLVERSION=gcl-2.6.10
```

This will cause Axiom to untar this `tgz` file to get the sources and apply the  
 corresponding patches

7. change the Makefile to match `Makefile.pamphlet`

- compile the tangle program

```
( cd books ; gcc -o tangle tangle.c )
```

- use `books/tangle` to extract the new Makefile

```
books/tangle Makefile.pamphlet >Makefile
```

## 0.8 Literate Programming

The Axiom source code was originally developed at IBM Research. It was sold to The Numerical Algorithms Group (NAG) and was on the market as a commercial competitor to Mathematica and Maple.

Axiom was withdrawn from the market in 2000 and released as free and open source software in 2001. When the Axiom project was started on savannah, the GNU Free Software Foundation site the source code had been rewritten into “pamphlet” files. The reasons for this are twofold.

### Pamphlet files

When the Axiom code was released it contained few comments. That made it very difficult to understand what the code actually did. Unlike commercial software there would be no group of individuals who would work on the project for its lifetime. Thus there needed to be a way to capture the expertise and understanding underlying ongoing development.

Unlike any other piece of free and open source software Axiom will still give useful answers 30 years from now. Thus it is important, and worthwhile, to invest a large amount of effort into documenting how these answers are arrived at and why the algorithms are written the way they are.

The pamphlet file format follows Knuth’s idea of literate programming. Knuth made the observation that a program should be a work designed to be read by humans. Making the program readable by machine was a secondary consideration. Making documentation primary and code secondary was a dramatic shift for a programmer.

Knuth created a file format that combined documentation and code. He created a tool called “Web” which had two basic command, tangle and weave. The tangle command would be run against a literate document and extract the source code, the weave command would be run against the literate document and extract the TeX.

### noweb

Knuth’s Web tool was specifically designed to use Pascal code. The “tangle” operation would prettyprint the output according to the style rules of Pascal.

Axiom was written in a variety of languages, such as C and Lisp, and used tools such as Makefiles which have their own syntax. Thus Web could not be used directly.

Axiom defines a new latex environment called chunk. This chunk environment makes the pamphlet file a pure latex file. This eliminates the need for the weave operation. The tangle operation only needs to occur while manipulating code, either during system build or end user interaction. At both of these times the tangle operation can be built into the system and hidden.

To support extracting chunks from pamphlet files Axiom now has a new top level command. At the top level one can write:

```
)tangle filename
```

This will look for “filename.pamphlet” and extract the top level chunk which has the name “\*”.

The latest changeset introduces two related changes, gclweb and axiom.sty. Together these changes allow optional syntactic changes to pamphlets. These changes will completely eliminate the need to weave files since now a pamphlet file can be a valid latex file. Tangle is the only remaining command and it will eventually be an option on )compile, etc.

The src/interp/gclweb.lisp file introduces the ability to extract code from pamphlet files while inside Axiom. The short description is that gclweb will now automatically distinguish the type of chunk style (latex or noweb) based on the chunk name. It is a first step to a native understanding of pamphlet files. Future work involves integrating it into commands like )compile and adding commands like )tangle.

Tangle can also be called directly from lisp on a file from within Axiom:

```
)lisp (tangle "filename.pamphlet" "chunkname")
      )lisp (tangle "filename.pamphlet" "chunkname" "filename.spad")
```

gclweb distinguishes the input syntax by looking at the first character of the chunkname. If it is a '<' then noweb is used, otherwise latex.

The src/doc/axiom.sty.pamphlet introduces the new chunk environment. This is a completely compatible change and has no impact on existing pamphlets. The new syntax makes pamphlet files = tex files so there is no need to use weave. The gclweb change has a compatible tangle function which can be invoked from inside Axiom.

```
\begin{chunk}{chunkname}
  your code goes here
\end{chunk}
```

One feature of the latex chunk style is that latex commands work within the chunk. To get typeset mathematics use \ ( and \)

```
-- This will typeset in a chunk \ ( x^2+\epsilon \)
-- And you can format things {\bf bold}
```

## 0.9 Databases

### libcheck

The databases are built from the .kaf files in the .nrlib directories. (.kaf files are random access files).

interp.exposed is a file that names all of the CDPs (Category, Domain, and Packages) and classifies them. Only some CDPs are exposed because most are used to implement algebra and are not intended to be user level functions. Exposing all of the functions causes much ambiguity.

There is a function called libcheck (see src/interp/util.lisp.pamphlet) that will check nrlibs vs interp.exposed. This is only partially functional as I see that changes were made to the system which broke this function.

The libcheck function requires an absolute pathname to the int directory so call it thus:

```
--> )lisp (libcheck "/axiom/int/algebra")
```

The main reason this function is broken is that the system now gets exposure information from src/algebra/exposed.lsp.pamphlet. It appears that interp.exposed.pamphlet is no longer used (although I made sure that both files have the same information). I'm going to

modify libcheck to use exposed.lsp in the future and eliminate all references in the system to interp.exposed.

For the moment, however, the libcheck function is quite useful. It used to be run during system build because I frequently ran into database problems and this function would alert me to that fact. I'll add it back into the Makefile once I elide interp.exposed.

## asq

Axiom has several databases which contain information about domains, categories, and packages. The databases are in a compressed format and are organized as random-access files using numeric index values so it is hard to get at the stored information. However, there is a command-line query function called asq (pronounced ask) that knows the format of the files and can be used for stand-alone queries. For instance, if you know the abbreviation for a domain but want to know what source file generated that domain you can say:

```
asq -so FOOBAR
```

and it will tell you the name of the algebra source file that defines FOOBAR.

## 0.10 Axiom internal representations

### PRIMITIVE REPRESENTATIONS OF AXIOM OBJECTS

There are several primitive representations in axiom. These are:

boolean

this is represented as a lisp boolean

integer

this is represented as a lisp integer

small integer

this is represented as a lisp integer

small float

this is represented as a lisp float

list

this is represented as a lisp list

vector

this is represented as a lisp vector

record

there are 3 cases:

records of 1 element are a pair (element . nil)

records of 2 element are a pair (element1 . element2)

records of 3 or more are a vectors #<a b c...>

mapping

mappings are a spadcall objects. they are represented as a pair  
 (lispfn . env)  
 where the env is usually a type object. A spadcall rips this  
 pair open and applies the lispfn to its args with env as the  
 last arg.

union

there are 2 cases

if the object can be determined by a lisp predicate  
 (eg integer) then the union is just the object (eg 3)  
 itself since we can use lisp to decide which branch of  
 the union the object belongs to. that is, 3 is of the  
 integer branch in union(list,integer)

if the object cannot be determined then the object is  
 wrapped into a pair where the car of the pair is the  
 union branch name and the cdr of the pair is the object.  
 that is, given union(a:SUP,b:POLY(INT)) x might be (a . x)

note: if no tags are given in the union the system uses  
 consecutive integers, thus union(SUP,POLY(INT)) will give  
 a pair of (1 . x) or (2 . x) depending on the type of x

other types are built up of compositions of these primitive  
 types. a sparse univariate polynomial (SUP) over the integers

$x^{**2}+1$

is represented as

```
Term := Record(k:NonNegativeInteger,c:R)
Rep  := List Term
```

that is, the representation is a list of terms where each term  
 is a record whose first field is a nonnegative integer (the  
 exponent) and the second field is a member of the coefficient  
 ring. since this is a record of length 2 it is represented as  
 a pair. thus, the internal form of this polynomial is:

((2 . 1) (0 . 1))

a more complex object (recursively defined) is POLY(INT). given

$x^{**2}+1$

as a POLY(INT) we look at its representation and see:

```
D := SparseUnivariatePolynomial($)
VPoly := Record(v:VarSet,ts:D)
Rep := Union(R,VPoly)
```

so first we find that we are a member of the second form of the  
 union and since this is an untagged union the system uses 2 as  
 the tag. thus the first level of internal representation is:

```
( 2 . <a VPoly object> )
```

next we need to define the VPoly object. VPolys are records of length 2 so we know they are represented by a pair. the car of the pair is a VarSet. the cdr is a D which is a SparseUnivariatePolynomial. Thus we consider this to be a poly in x (at the top level) and we get:

```
( 2 . ( x . <an SUP>))
```

the SUP is over the SparseMultivariatePolynomials (SMP) so the representation is recursive. Since an SUP is represented as a list of

```
(non-negative int . coefficient)
```

one per term and we have 2 terms we know the next level of structure is:

```
( 2 . ( x . (( 2 . <an SMP> ) ( 0 . <an SMP> ))))
```

the SMP is just the integers so it fits into the first branch of the union and each SMP looks like:

```
( uniontag . value )
```

in this case, being the first branch we get

```
( 2 . ( x . (( 2 . ( 1 . 1 )) ( 0 . ( 1 . 1 )))))
```

as the internal representation of

```
x**2 + 1
```

what could be easier?

## 0.11 Spad to internal function calling

### getdatabase output

```
GETDATABASE('Permutation, 'OPERATIONALIST)$Lisp
```

generates the output

```
((($unique)
 (~= (((Boolean) $ $) () T ELT))
 (sort (((List $) (List $)) 76 T ELT))
 (sign (((Integer) $) 59 T ELT))
 (sample (($) () T CONST))
 (recip (((Union $ "failed") $) () T ELT))
 (order (((NonNegativeInteger) $) 57 T ELT))
 (orbit (((Set #1) $ #1) 48 T ELT))
```

```

(one? (((Boolean) $) () T ELT))
(odd? (((Boolean) $) 62 T ELT))
(numberOfCycles (((NonNegativeInteger) $) 60 T ELT))
(movedPoints (((Set #1) $) 41 T ELT))
(min (($ $ $) () (OR (has #1 (Finite)) (has #1 (OrderedSet)))) ELT))
(max (($ $ $) () (OR (has #1 (Finite)) (has #1 (OrderedSet)))) ELT))
(listRepresentation
  ((Record (: preimage (List #1)) (: image (List #1))) $) 35 T ELT))
(latex (((String) $) () T ELT))
(inv (($ $) 92 T ELT))
(hash (((SingleInteger) $) () T ELT))
(fixedPoints (((Set #1) $) 98 (has #1 (Finite)) ELT))
(even? (((Boolean) $) 58 T ELT))
(eval ((#1 $ #1) 46 T ELT))
(elt ((#1 $ #1) 93 T ELT))
(degree (((NonNegativeInteger) $) 43 T ELT))
(cycles (($ (List (List #1))) 84 T ELT))
(cyclePartition (((Partition) $) 52 T ELT))
(cycle (($ (List #1)) 21 T ELT))
(conjugate (($ $ $) () T ELT))
(commutator (($ $ $) () T ELT))
(coercePreimagesImages (($ (List (List #1))) 38 T ELT))
(coerceListOfPairs (($ (List (List #1))) 87 T ELT))
(coerceImages (($ (List #1)) 95 T ELT))
(coerce (((OutputForm) $) 83 T ELT) (($ (List (List #1))) 65 T ELT)
  (($ (List #1)) 66 T ELT))
(^ (($ $ (PositiveInteger)) () T ELT)
  (($ $ (NonNegativeInteger)) () T ELT) (($ $ (Integer)) () T ELT))
(One (($) 16 T CONST))
(>= (((Boolean) $ $) () (OR (has #1 (Finite)) (has #1 (OrderedSet)))) ELT))
(> (((Boolean) $ $) () (OR (has #1 (Finite)) (has #1 (OrderedSet)))) ELT))
(= (((Boolean) $ $) 44 T ELT))
(<= (((Boolean) $ $) () (OR (has #1 (Finite)) (has #1 (OrderedSet)))) ELT))
(< (((Boolean) $ $) 64 T ELT))
(/ (($ $ $) () T ELT))
(** (($ $ (PositiveInteger)) () T ELT)
  (($ $ (NonNegativeInteger)) () T ELT) (($ $ (Integer)) () T ELT))
(* (($ $ $) 22 T ELT))

```

Sometimes in a getdatabase expression you will see:

```

(~= (((Boolean) $ $) () T ELT))
-----^^

```

and in other places there is a number

```

(sign (((Integer) $) 59 T ELT))
-----^^

```

In general, when a large number appears it is a byte index into the compress.daase file.

Axiom would not fit on a laptop. We needed smaller databases. The solution to the problem was to scan the datatases for common substrings, write the substring to compress.daase, and replace the substring by the byte offset.

When reading the database these numbers would be replaced by the substring from compress.daase using random access seeks based on the byte offset.



See book volume 5 for an explanation of the database file formats.

HOWEVER, in this case, the number has a different meaning which I will talk about below.

In summary, this shows what the following incantation means:

```
(sign (((Integer) $) () (has #1 (OrderedIntegralDomain))))
INTEGER inherits sign from OINTDOM (OrderedIntegralDomain)
OINTDOM inherits sign from ORDRING (OrderedRing)
ORDRING implements sign
  since ORDRING is a category, the actual code lives in
  ORDRING-.nrllib/code.lsp
```

The code for sign in ORDRING-.nrllib/code.lsp has the signature:

```
(DEFUN |ORDRING-;sign;SI;3| (|x| $) ....)
```

We can "decode" the meaning of the function name as

- **ORDRING-** the implementing file
- **sign** the function name
- **SI** returns SingleInteger (an old domain name)
- **3** the third function in the file (unique, to distinguish multiple functions with the same name)

It takes 2 arguments,

- **|x|** which should be a SingleInteger
- **\$** which is the current domain (ORDRING-)

So it looks like I have the following structure

```
(NAME ((TARGETTYPE SOURCETYPE) ?1 CONDITION ?2))
```

but we are looking up 'sign' in INTEGER so there is a condition on sign

```
Integer has OrderedRing ==> true
```

so that explains the condition field.

Here we show how Axiom finds the function implementation, looks up the function "in the domain", and calls it.

```
(sign (((Integer) $) 59 T ELT))
```

Now you've asked for 'sign' from domain Permutation

```
(sign (((Integer) $) 59 T ELT))
```

The implementation for 'sign' is in PERM.nrllib/code.lsp. It reads:

```
(defun |PERM;sign;$I;17| (|p| $)
  (cond
    ((spadcall |p| (qrefelt $ 58)) 1)
    ('t -1)))
```

which you would read as

```
if (calling function in position 58 of myself) is true
  then return 1
  else return -1
```

How does Axiom find the function? It is in the `infovec` which is the “information vector” containing information about the domain.

First we must make sure that PERM has the necessary domain information loaded (the ‘`infovec`’).

```
-> [1,2,3]::PERM(INT)
```

Now, back to the ‘`sign`’ function. You saw this:

```
(sign (((Integer) $) 59 T ELT)) (sample (($) () T CONST))
```

which is asking you to look up element 59 from the domain (`$`)

Note that `$` *actually* means the `infovec`. So we are asking:

```
(elt (elt (getf (symbol-plist 'Permutation) 'infovec) 0) 59)
```

which results in:

```
|PERM;sign;$I;17|
```

so we “looked up” the function `sign` in the domain PERM.

Explaining in more detail, from the inside out by walking the runtime data structures we see

```
(symbol-plist 'Permutation)
```

returns the property list on the symbol **Permutation** which is where Axiom caches domain information. Almost everything of interest about a domain resides on the property list, shown here in all its glory.

```
(LOADED "/research/test/mnt/ubuntu/algebra/PERM.o"
SYSTEM:DEBUG (:G1567 #:G1568)
|infovec| (
#(NIL NIL NIL NIL NIL
(|local| |#1|) (QUOTE |Rep|) (|Boolean|)
(0 . <) (|PositiveInteger|) (6 . |lookup|)
(|Integer|) (|List| 6) (11 . |maxIndex|)
(16 . |elt|)
(CONS IDENTITY (FUNCALL (|dispatchFunction| |PERM;One;$;29|) $))
(|NonNegativeInteger|) (22 . |last|) (28 . |first|)
(34 . |concat|) |PERM;cycle;L$;26| |PERM;*;3$;28|
(40 . =) (46 . =) (52 . |elt|)
(58 . |list|) (63 . |position|) (69 . |delete|)
(|Mapping| 8 13 13) (|List| 13) (75 . |sort|)
(81 . |copy|) (86 . |member?|)
(|Record| (|:| |preimage| 13) (|:| |image| 13))
|PERM;listRepresentation;$R;9| (92 . |elt|)
(98 . ~|=) |PERM;coercePreimagesImages;L$;10|
(|Set| 6) (104 . |construct|) |PERM;movedPoints;$S;11|
(109 . |#|) |PERM;degree;$Nni;12| |PERM;=;2$B;13|
(114 . |brace|) |PERM;eval;$2S;31| (119 . |insert!|)
|PERM;orbit;$SS;14| (|List| 12) (|Partition|)
(125 . |partition|) |PERM;cyclePartition;$P;15|
(130 . |convert|) (135 . |removeDuplicates|)
(|List| $) (140 . |lcm|) |PERM;order;$Nni;16|
|PERM;even?;$B;18| |PERM;sign;$I;17| |PERM;numberOfCycles;$Nni;33|
(145 . |even?|) |PERM;odd?;$B;19| (150 . |maxIndex|)
|PERM;<;2$B;20| |PERM;coerce;L$;21| |PERM;coerce;L$;22|
```

```

(|Record| (|:| |cycl| 30) (|:| |permut| $$))
(|List| 67) (155 . |cons|) (|Mapping| 8 67 67)
(161 . |sort|) (|List| $$) (167 . |nil|)
(171 . |cons|) (177 . |reverse|) |PERM;sort;2L;23|
(|OutputForm|) (182 . |coerce|) (187 . |blankSeparate|)
(192 . |paren|) (197 . |outputForm|) (202 . |hconcat|)
|PERM;coerce;$0f;24| |PERM;cycles;L$;25| (207 . |second|)
(212 . =) |PERM;coerceListOfPairs;L$;27|
(|Vector| 6) (218 . |construct|) (223 . |elt|)
(229 . |new|) |PERM;inv;2$;30| |PERM;elt;$2S;32|
(235 . |coerce|) (240 . |coerceImages|) (245 . |index|)
(250 . |complement|) (255 . |fixedPoints|) (260 . |conjugate|)
(265 . +) (|Union| $ (QUOTE "failed"))
(|SingleInteger|) (|String|)
#(~= 271 |sort| 277 |sign| 282
|sample| 287 |recip| 291 |order| 296
|orbit| 301 |one?| 307 |odd?| 312
|numberOfCycles| 317 |movedPoints| 322 |min| 327
|max| 333 |listRepresentation| 339 |latex| 344
|inv| 349 |hash| 354 |fixedPoints| 359
|even?| 364 |eval| 369 |elt| 375
|degree| 381 |cycles| 386 |cyclePartition| 391
|cycle| 396 |conjugate| 401 |commutator| 407
|coercePreimagesImages| 413 |coerceListOfPairs| 418 |coerceImages| 423
|coerce| 428 ^ 443 |One| 461
>= 465 > 471 = 477
<= 483 < 489 / 495
** 501 * 519)
((|unitsKnown| . 0))
#(0 0 0 0 3 0 0 0)
#(NIL
|Group&|
|Monoid&|
|SemiGroup&|
|OrderedSet&|
|SetCategory&|
|BasicType&|
NIL)
#((|PermutationCategory| 6)
(|Group|)
(|Monoid|)
(|SemiGroup|)
(|OrderedSet|)
(|SetCategory|)
(|BasicType|)
(|CoercibleTo| 77))
.
#( 2 6 8 0 0 9 1 6 10 0 11 1 13 12 0 14 2 13 6 0 12
15 2 13 0 0 17 18 2 13 0 0 17 19 2 13 0 0 0 20 2 6
8 0 0 23 2 13 8 0 0 24 2 7 13 0 12 25 1 13 0 6 26
2 13 12 6 0 27 2 13 0 0 12 28 2 30 0 29 0 31 1 13 0
0 32 2 13 8 6 0 33 2 30 13 0 12 36 2 6 8 0 0 37 1
39 0 13 40 1 39 17 0 42 1 39 0 13 45 2 39 0 6 0 47 1
50 0 49 51 1 50 49 0 53 1 49 0 0 54 1 12 0 55 56 1 12

```

```

8 0 61 1 30 12 0 63 2 68 0 67 0 69 2 68 0 70 0 71 0
72 0 73 2 72 0 2 0 74 1 72 0 0 75 1 6 77 0 78 1 77
0 55 79 1 77 0 0 80 1 77 0 12 81 1 77 0 55 82 1 13 6
0 85 2 39 8 0 0 86 1 88 0 13 89 2 88 6 0 12 90 2 7
0 17 13 91 1 6 0 12 94 1 0 0 13 95 1 6 0 10 96 1 39
0 0 97 1 0 39 0 98 1 50 0 0 99 2 50 0 0 0 100 2 0
8 0 0 1 1 0 55 55 76 1 0 12 0 59 0 0 0 1 1 0 101
0 1 1 0 17 0 57 2 0 39 0 6 48 1 0 8 0 1 1 0 8
0 62 1 0 17 0 60 1 0 39 0 41 2 3 0 0 0 1 2 3 0
0 0 1 1 0 34 0 35 1 0 103 0 1 1 0 0 0 92 1 0 102
0 1 1 1 39 0 98 1 0 8 0 58 2 0 6 0 6 46 2 0 6
0 6 93 1 0 17 0 43 1 0 0 30 84 1 0 50 0 52 1 0 0
13 21 2 0 0 0 0 1 2 0 0 0 0 1 1 0 0 30 38 1 0
0 30 87 1 0 0 13 95 1 0 0 13 66 1 0 0 30 65 1 0 77
0 83 2 0 0 0 12 1 2 0 0 0 17 1 2 0 0 0 10 1 0
0 0 16 2 3 8 0 0 1 2 3 8 0 0 1 2 0 8 0 0 44
2 3 8 0 0 1 2 0 8 0 0 64 2 0 0 0 0 1 2 0 0
0 12 1 2 0 0 0 17 1 2 0 0 0 10 1 2 0 0 0 0 22))
|lookupComplete|)
PNAME "Permutation"
DATABASE
#S(DATABASE
  ABBREVIATION PERM
  ANCESTORS NIL
  CONSTRUCTOR NIL
  CONSTRUCTORCATEGORY 2444459
  CONSTRUCTORKIND |domain|
  CONSTRUCTORMODEMAP
  (((|Permutation| |#1|)
    (|Join|
      (|PermutationCategory| |#1|)
      (CATEGORY |domain|
        (SIGNATURE |listRepresentation|
          ((|Record| (|:| |preimage| #) (|:| |image| #)) $))
          (SIGNATURE |coercePreimagesImages| ($ (|List| (|List| |#1|))))
          (SIGNATURE |coerce| ($ (|List| (|List| |#1|))))
          (SIGNATURE |coerce| ($ (|List| |#1|)))
          (SIGNATURE |coerceListOfPairs| ($ (|List| (|List| |#1|))))
          (SIGNATURE |degree| ((|NonNegativeInteger|) $))
          (SIGNATURE |movedPoints| ((|Set| |#1|) $))
          (SIGNATURE |cyclePartition| ((|Partition|) $))
          (SIGNATURE |order| ((|NonNegativeInteger|) $))
          (SIGNATURE |numberOfCycles| ((|NonNegativeInteger|) $))
          (SIGNATURE |sign| ((|Integer|) $))
          (SIGNATURE |even?| ((|Boolean|) $))
          (SIGNATURE |odd?| ((|Boolean|) $))
          (SIGNATURE |sort| ((|List| $) (|List| $)))
          (IF (|has| |#1| (|Finite|))
            (SIGNATURE |fixedPoints| ((|Set| |#1|) $) |noBranch|)
            (IF (|has| |#1| (|IntegerNumberSystem|))
              (SIGNATURE |coerceImages| ($ (|List| |#1|)))
              (IF (|has| |#1| (|Finite|))
                (SIGNATURE |coerceImages| ($ #) |noBranch|))))
            (|SetCategory|))

```

```

(T |Permutation|))
COSIG (NIL T)
DEFAULTDOMAIN NIL
MODEMAPS 2443154
NILADIC NIL
OBJECT "PERM"
OPERATIONALIST
((|$unique|)
 (~= (((|Boolean|) $ $) NIL . #0=(T . #1=(ELT))))
 (|sort| (((|List| $) (|List| $)) 76 . #0#))
 (|sign| (((|Integer|) $) 59 . #0#))
 (|sample| (($ NIL T CONST))
 (|recip| (((|Union| $ "failed") $) NIL . #0#))
 (|order| (((|NonNegativeInteger|) $) 57 . #0#))
 (|orbit| (((|Set| |#1|) $ |#1|) 48 . #0#))
 (|one?| (((|Boolean|) $) NIL . #0#))
 (|odd?| (((|Boolean|) $) 62 . #0#))
 (|numberOfCycles| (((|NonNegativeInteger|) $) 60 . #0#))
 (|movedPoints| (((|Set| |#1|) $) 41 . #0#))
 (|min| (($ $ $) NIL
 (OR (|has| |#1| (|Finite|)) (|has| |#1| (|OrderedSet|))) . #1#))
 (|max| (($ $ $) NIL
 (OR (|has| |#1| (|Finite|)) (|has| |#1| (|OrderedSet|))) . #1#))
 (|listRepresentation|
 (((|Record| (|:| |preimage| (|List| |#1|))
 (|:| |image| (|List| |#1|))) $) 35 . #0#))
 (|latex| (((|String|) $) NIL . #0#))
 (|inv| (($ $) 92 . #0#))
 (|hash| (((|SingleInteger|) $) NIL . #0#))
 (|fixedPoints| (((|Set| |#1|) $) 98 (|has| |#1| (|Finite|)) . #1#))
 (|even?| (((|Boolean|) $) 58 . #0#))
 (|eval| ((|#1| $ |#1|) 46 . #0#))
 (|elt| ((|#1| $ |#1|) 93 . #0#))
 (|degree| (((|NonNegativeInteger|) $) 43 . #0#))
 (|cycles| (($ (|List| (|List| |#1|))) 84 . #0#))
 (|cyclePartition| (((|Partition|) $) 52 . #0#))
 (|cycle| (($ (|List| |#1|)) 21 . #0#))
 (|conjugate| (($ $ $) NIL . #0#))
 (|commutator| (($ $ $) NIL . #0#))
 (|coercePreimagesImages| (($ (|List| (|List| |#1|))) 38 . #0#))
 (|coerceListOfPairs| (($ (|List| (|List| |#1|))) 87 . #0#))
 (|coerceImages| (($ (|List| |#1|)) 95 . #0#))
 (|coerce|
 (((|OutputForm|) $) 83 . #0#)
 (($ (|List| (|List| |#1|))) 65 . #0#)
 (($ (|List| |#1|)) 66 . #0#))
 (^ (($ $ (|PositiveInteger|)) NIL . #0#)
 (($ $ (|NonNegativeInteger|)) NIL . #0#)
 (($ $ (|Integer|)) NIL . #0#))
 (|One| (($) 16 T CONST))
 (>= (((|Boolean|) $ $) NIL
 (OR (|has| |#1| (|Finite|)) (|has| |#1| (|OrderedSet|))) . #1#))
 (> (((|Boolean|) $ $) NIL
 (OR (|has| |#1| (|Finite|)) (|has| |#1| (|OrderedSet|))) . #1#))

```

```

(= (((|Boolean|) $ $) 44 . #0#))
(<= (((|Boolean|) $ $) NIL
      (OR (|has| |#1| (|Finite|)) (|has| |#1| (|OrderedSet|))) . #1#))
(< (((|Boolean|) $ $) 64 . #0#))
(/ (($ $ $) NIL . #0#))
(** (($ $ (|PositiveInteger|)) NIL . #0#)
     (($ $ (|NonNegativeInteger|)) NIL . #0#)
     (($ $ (|Integer|)) NIL . #0#))
(* (($ $ $) 22 . #0#))
DOCUMENTATION 1609893
CONSTRUCTORFORM 1609883
ATTRIBUTES 1614391
PREDICATES 1614406
SOURCEFILE "bookvol10.3.pamphlet"
PARENTS NIL
USERS NIL
DEPENDENTS NIL
SPARE NIL))

```

There are many things on the property list which looks like

```
(symbol1 thing1 symbol2 thing2 ... symboln thingn)
```

In the PERM case we see

```
(LOADED "/research/silver/mnt/algebra/PERM.o"
 |infovec| (#<vector> #<vector>...)
 ....)
```

We can get the `|infovec|` off the property list with the call

```
(getf (symbol-plist '—Permutation—) '—infovec—)
```

is a request to search the property list for the symbol `—infovec—` and return the value, which is the domain "information vector".

You can see this vector if you look in `PERM.nrlib/code.lsp`. At the bottom of that file you'll see:

```
(SETF (GET (QUOTE |Permutation|) (QUOTE |infovec|) ....)
```

which uses `SETF` to put the `infovec` on the property list of `PERM`. This information vector contains information for function lookup. This vector gets created when we "instantiate" `PERM`.

The `infovec` is a list with the structure

```
(#<vector 08ea516c>
 #<vector 08ea5150>
 ((|unitsKnown| . 0))
 (#<vector 08ea50fc>
 #<vector 08ea5134>
 #<vector 08ea5118> . #<vector 08ea50e0>)
 |lookupComplete|)
```

So, now that we have the `infovec`, back to the game...

```
(elt (getf (symbol-plist '|Permutation|) '|infovec|) 0)
```

This gets the 0th element out of the `infovec` list which is a vector of the name of every function `Permutation` implements. We look up function names in this list, in particular, 59:

```
(elt (elt (getf (symbol-plist '|Permutation|) '|infovec|) 0) 59)
```

looks into this vector of names at the 59th element which returns

```
|PERM;sign;$I;17|
```

The SPAD form of this function reads:

```
sign(p) ==
  even? p => 1
  -1
```

The lisp form (see PERM.nrlib/code.lsp) reads:

```
(defun |PERM;sign;$I;17| (|p| $)
  (cond
    ((spadcall |p| (qrefelt $ 58)) 1)
    ('t -1)))
```

We call the `|PERM;sign;$I;17|` which takes 2 arguments

The first of which is the permutation and the second is the infovec for the PERM domain.

The `(qrefelt $ 58)` uses the above dance to look up a function in the infovec at the 58th position... which returns

```
|PERM;even?;$B;18|
```

The `spadcall` calls `|PERM;even?;$B;18|` with the value of `|p|`.

If we look in the domain `Permutation` for the implementation of `even?`

```
even?(p) == even?(#(p.1) - numberOfCycles p)
```

which in PERM.nrlib/code.lsp we see

```
(defun |PERM;even?;$b;18|
  (spadcall
    (- (length (spadcall |p| 1 (qrefelt $ 25)))
      (spadcall |p| (qrefelt $ 60)))
    (qrefelt % 61)))
```

where

```
(qrefelt $ 25) ==> (52 . |elt|)
(qrefelt $ 60) ==> |PERM;numberOfCycles;$Nni;33|
(qrefelt $ 61) ==> (145. |even?|)
```

So, to summarize, the small magic numbers you see in the results are indexes into the infovec, which is where Axiom stores things it needs to look up at runtime, usually function references.

If there is `()` rather than a number then there is no need to do a function lookup.

Axiom execution is an alternating series of function lookups in the infovec followed by a call of that function which results in a function lookup in the infovec followed by a call of that function which results in ....

`spadcall` is a wrapper macro which takes the arguments and a function to call. `qrefelt` does the infovec lookup.

## 0.12 axiom command

The axiom command will eventually be a shell script. At the moment it is just a copy of the intersys image. However the whole Axiom system consists of several processes and the axiom command starts these processes. The shell script will transparently replace the axiom executable image which will be renamed to spadsys.

## 0.13 help command documentation

Axiom supports a `)help` command that takes a single argument. This argument is interpreted as the name of a flat ascii file which should live in `$AXIOM/doc/src/spadhelp`.

### help documentation for algebra

The help documentation for algebra files lives within the algebra pamphlet. The help chunk contains the name of the domain, thus:

```
\begin{chunk}{thisdomain.help}
=====
thisdomain examples
=====

      (documentation for this domain)

examplefunction foo
  output
          Type: thetype

See Also:
o )show thisdomain
o $AXIOM/bin/src/doc/algebra/thisfile.spad.dvi

\end{chunk}
```

The documentation starts off with the domain enclosed in two lines of equal signs. The documentation is free format. Generally the functions are indented two spaces, the output is indented 3 spaces, and the Type field has been moved toward the center of the line.

The “See Also:” section lists the domain with the “show” command and the path to the source file in dvi format.

### Adding help documentation in Makefile

There is a section in the `src/algebra/Makefile.pamphlet` that reads:

```
SPADHELP=\
  ${HELP}/AssociationList.help  ${HELP}/BalancedBinaryTree.help \
```

which is essentially a list of all of the algebra help files. Each item in this list refers to a stanza that looks like:

```
${HELP}/AssociationList.help: ${BOOKS}/bookvol10.3.pamphlet
```



```

@echo 7000 create AssociationList.help from \
    ${BOOKS}/bookvol10.3.pamphlet
@${TANGLE} -R"AssociationList.help" ${BOOKS}/bookvol10.3.pamphlet \
    >${HELP}/AssociationList.help
@cp ${HELP}/AssociationList.help ${HELP}/ALIST.help
@${TANGLE} -R"AssociationList.input" ${BOOKS}/bookvol10.3.pamphlet \
    >${INPUT}/AssociationList.input
@echo "AssociationList (ALIST)" >>${HELPPFILE}

```

Notice that the first line has an connection between the help file and the spad file that contains it.

The second line gives debugging output containing a unique number for console debugging purposes of failed builds.

The third line extracts the help file. Help files are part of the algebra books (bookvol10.2, bookvol10.3, and bookvol10.4). The chunkname is the same as the Category, Domain, or Package.

The fourth line copies the file with the long name of the domain to a file with the abbreviation of the domain so the user can query the domain with either form using help.

The fifth line creates a regression test file for the help file. In the algebra each help file has an associated regression test file to test all of the function calls shown in the help page. These files are copied to the intermediate directory for regression testing.

The sixth line adds a line to the HELPPFILE (see the variable in the src/algebra/Makefile). This HELPPFILE is concatenated onto the final help.help file in the MNT/doc/spadhelp directory. Thus, when a user types )help with no argument they see a list of domains which contain help information.

## Using help documentation for regression testing

The fifth line extracts an input test file for the algebra. In general each help file is used to create an input test file for regression testing.

There is a Makefile variable called REGRESS in the algebra Makefile:

```

REGRESS=\
  AssociationList.regress  BalancedBinaryTree.regress \

```

This is part of a Makefile that structure within the algebra Makefile. This Makefile gets extracted by the Makefile in the input subdirectory. Thus there is a connection between the two Makefiles (algebra and input). This algebra regression Makefile goes by the chunk name **algebra.regress**. It contains a list of regression files and a single stanza:

```

%.regress: %.input
@ echo algebra regression testing $*
@ rm -f $*.output
@ echo ')read $*.input' | ${TESTSYS}
@ echo ')lisp (regress "$*.output")' | ${TESTSYS} \
    | egrep -v '(Timestamp|Version)' | tee $*.regress

```

The input Makefile extracts **algebra.regress** and then calls make to process this file.

This keeps the regression test list in the algebra Makefile.

## help documentation as algebra test files

### 0.14 debugsys

The “debugsys” executable is the “interpsys” image but it is built using the interpreted lisp code rather than using compiled lisp code. This will make it slower but may, in certain cases, give much better feedback in case of errors. If you find you need to use debugsys you’re really doing deep debugging. It isn’t useful for much else. It can be started by typing:

```
export AXIOM=/home/axiomgnu/new/mnt/linux
/home/axiomgnu/new/obj/linux/bin/debugpsys
```

Notice that this image lives in the “obj” subtree. It is not shipped with the “final” system image as only developers could find it useful.

#### debugging hyperdoc

Hyperdoc will sometimes exit and also kill the AXIOMsys image with no error message. One way to get around this is to replace the AXIOMsys image with the debugsys image:

1. mv \$AXIOM/bin/AXIOMsys \$AXIOM/bin/AXIOMsys.backup  
This keeps the failing axiomsys image around for later restoration.
2. cp obj/sys/bin/debugsys \$AXIOM/bin/AXIOMsys  
This puts an interpreted version of axiom in place of the compiled form
3. axiom  
Now we are running a fully interpreted form and the error messages are much more informative.

### 0.15 Understanding a compiled function

Suppose we stop a program at a function call to some low level lisp function, say ONEP. We can do that by entering

```
)trace ONEP )break
```

at the Axiom command prompt. Or at the lisp prompt:

```
(trace (ONEP :entry (break)))
```

Next we execute some function that will eventually call ONEP thus:

```
p := numeric %pi
```

```
Break: onep
Broken at ONEP. Type :H for Help.
BOOT>>
```

We have stopped and entered a lisp command prompt. We can enter any lisp expression here and there are commands that begin with a “:” character. “:b” requests a backtrace of the call stack, thus:

```
BOOT>>:b
Backtrace: funcall > system:top-level > restart > /read >
|upLET| > eval > |Pi| > |newGoGet| > |newGoGet| > ONEP
```

Here we see that the function `ONEP` was called by the function `newGoGet`. Notice that the name is surrounded by vertical bars. Vertical bars are a common lisp escape sequence used to allow non-standard characters to occur in symbol names. Common lisp is not case sensitive. Boot code is case sensitive. Thus symbol names that were written in Boot tend to have escape sequence characters around the name.

Now that we see the simple backtrace we can ask for a more complex one. The command is `“:bt”`. It shows more detail about each level of call on the invocation history stack (`ihs`) including the function name, its arguments and the depth of the invocation history stack (`[ihs=13]`):

```
BOOT>>:bt

#0  ONEP {1=nil,} [ihs=13]
#1  newGoGet {g3629="(0" (#<vector 08b34bb4> 45 . |char|)),
      loc1=#<compiled-function |CHAR;cha...} [ihs=12]
#2  newGoGet {g3629="(pi" (#<vector 08b34bec> 0 . |coerce|)),
      loc1=#<vector 08b34bec> 0 . |c...} [ihs=11]
#3  Pi {g109299=nil,loc1=nil,loc2=#<hash-table 082992f4>,
      loc3=|Pi|,loc4=15,loc5=#<vecto...} [ihs=10]
#4  EVAL {loc0=nil,loc1=nil,loc2=nil,
      loc3=#<compiled-function |Pi|>} [ihs=9]
#5  upLET {t=(#<vector 08b34d04> #<vector 08b34ce8>
      (#<vector 08b34ccc> (#<vector 08b34c08...} [ihs=8]
#6  /READ {loc0=#p"/home/axiomgnu/new/src/input/algbrbf.input",
      loc1=nil,loc2=nil,loc3=nil,...} [ihs=7]
#7  RESTART {loc0=(|read|
      |/home/axiomgnu/new/src/input/algbrbf.input|)),
      loc1=|/home/axiomg...} [ihs=6]
#8  TOP-LEVEL {loc0=nil,loc1=0,loc2=0,loc3=nil,loc4=nil,
      loc5=nil,loc6=nil,loc7=nil,loc8=nil,lo...} [ihs=5]
#9  FUNCCALL {loc0=#<compiled-function system:top-level>} [ihs=4]
BOOT>>:bl
>> (LAMBDA-BLOCK ONEP (&REST X) ...)():
X      : (1)
NIL
```

We can ask to see the local variables that are used at the current level of the invocation history stack. The command is `“:bl”` thus:

```
BOOT>>:bl
>> (LAMBDA-BLOCK ONEP (&REST X) ...)():
X      : (1)
NIL
```

We can move up the stack one level at a time looking at the function that called the current function (the previous function) using `“:p”` thus:

```
BOOT>>:p
Broken at |NEWGOGET|.
```

And again, we can look at the variables that can be accessed locally:

```
BOOT>>:bl
>> newGoGet():
Local0(G3629): (0 (#<vector 08b34bb4> 45 . char))
Local(1): #<compiled-function CHAR;char;S$;20>
Local(2): 0
```

```
Local(3): #<vector 08b233f0>
Local(4): 1
NIL
```

Here we see that the function `newGoGet` is calling `CHAR;char;S$;20` which is a mangled form of the name of the original `spad` function. To decode this name we can see that the `CHAR` portion is used to identify the domain where the function lives. This domain, `CHAR`, comes from the source file “string.spad” which lives in “src/algebra/string.spad.pamphlet”. To discover this we use the Axiom “`asq`” command with the “-so” (sourcefile) option at a standard shell prompt (NOT in the lisp prompt) thus:

```
asq -so CHAR
string.spad
```

If we look at the code in the `string.spad.pamphlet` file we find the following code signature:

```
char: String -> %
++ char(s) provides a character from a string s of length one.
```

and it’s implementation code:

```
char(s:String) ==
  (#s) = 1 => s(minIndex s) pretend %
  error "String is not a single character"
```

The `string.spad` file can be compiled at the command prompt. In particular, we can compile only the `CHAR` domain out of this file thus:

```
)co string.spad )con CHAR
```

This will produce a directory called `CHAR.NRLIB` containing 3 files:

```
ls CHAR.NRLIB
code.lsp index.kaf info
```

The `info` file contains information used by the `spad` compiler. We can ignore it for now.

The `index.kaf` file contains information that will go into the various Axiom database (.daase) files. The `kaf` file format is a random access file. The first entry is an integer that will be an index into the file that can be used in an operating system call to seek. In this case it will be an index which is the last used byte in the file. Go to the last expression in the file and we find:

```
(
  ("slot1Info" 0 11302)
  ("documentation" 0 9179)
  ("ancestors" 0 9036)
  ("parents" 0 9010)
  ("abbreviation" 0 9005)
  ("predicates" 0 NIL)
  ("attributes" 0 NIL)
  ("signaturesAndLocals" 0 8156)
  ("superDomain" 0 NIL)
  ("operationAlist" 0 7207)
  ("modemaps" 0 6037)
  ("sourceFile" 0 5994)
  ("constructorCategory" 0 5434)
  ("constructorModemap" 0 4840)
  ("constructorKind" 0 4831)
  ("constructorForm" 0 4817)
```

```
("NILADIC" 0 4768)
("compilerInfo" 0 2093)
("loadTimeStuff" 0 20))
```

This is a list of triples. Each triple has two interesting parts, the name of the data and the seek index of the data in the index.kaf file. So, for instance, if you want to know what source file contains this domain you can start at the top of the index.kaf file, move ahead 5994 bytes and you will be at the start of the string:

```
"/usr/local/axiom/src/algebra/string.spad"
```

The information in the index.kaf files are collected into the special databases (the .daase files). The stand-alone “asq” function can query these databases and answer questions. The kind of questions you can ask are the names in the list above.

The third file in the CHAR.NRLIB directory is the code.lsp file. This is the actual common lisp code that will be executed as a result of calling the various spad functions. The spad code from the char command was:

```
char(s:String) ==
  (#s) = 1 => s(minIndex s) pretend %
  error "String is not a single character"
```

which got compiled into the common lisp code:

```
(DEFUN |CHAR;char;S$;20| (|s| |$|)
  (COND
    ((EQL (QCSIZE |s|) 1)
      (SPADCALL |s|
        (SPADCALL |s| (QREFELT |$| 47))
        (QREFELT |$| 48)))
    ((QUOTE T)
      (|error| "String is not a single character"))))
```

To understand what is going on here we need to understand the low level details of Axiom’s interface to Common Lisp. The “Q” functions are strongly typed (Quick) versions of standard common lisp functions. QCSIZE is defined in src/interp/vmlisp.lisp.pamphlet thus:

```
(defmacro qcsize (x)
  '(the fixnum (length (the simple-string ,x))))
```

This macro will compute the length of a string.

QREFELT is defined in the same file as:

```
(defmacro qrefelt (vec ind)
  '(svref ,vec ,ind))
```

This macro will return the element of a vector.

SPADCALL is defined in src/interp/macros.lisp.pamphlet as:

```
(defmacro SPADCALL (&rest L)
  (let ((args (butlast L)) (fn (car (last L))) (gi (gensym)))
    '(let ((,gi ,fn)
      (the (values t) (funcall (car ,gi) ,@args (cdr ,gi))))
    ))
```

This macro will call the last value of the argument list as a function and give it everything but the last argument as arguments to the function. There are confusing historical reasons for this I won’t go into here.

So you can see that these are simply macros that will expand into highly optimizable (the optimizations depend on the abilities of the common lisp compiler) common lisp code.

The common lisp code computes the length of the string `s`. If the length is 1 then we call the `minIndex` function from `string` on `s`. The `minIndex` function is found by looking “in the domain”. The compiler changes the `minIndex` function call into a reference into a vector. The 47th element of the vector contains the function `minIndex`.

```
(SPADCALL |s| (QREFELT |$| 47))
```

This code is equivalent (ignoring the gensyms) to the call

```
(minIndex s)
```

The `$` symbol refers to the domain. At runtime this amounts to a lookup of the “infunc”. The compile-time infunc shown here:

```
(setf (get
  (QUOTE |Character|)
  (QUOTE |infunc|))
  (LIST
    (QUOTE
      #NIL
      NIL
      NIL
      NIL
      NIL
      NIL
      (QUOTE |Rep|)
      (|List| 28)
      (|PrimitiveArray| 28)
      (0 . |construct|)
      (QUOTE |OutChars|)
      (QUOTE |minChar|)
      (|Boolean|)
      |CHAR;=;2$B;1|
      |CHAR;<;2$B;2|
      (|NonNegativeInteger|)
      |CHAR;size;Nni;3|
      (|Integer|)
      |CHAR;char;I$;6|
      (|PositiveInteger|)
      |CHAR;index;Pi$;4|
      |CHAR;ord;$I;7|
      |CHAR;lookup;$Pi;5|
      (5 . |coerce|)
      |CHAR;random;$;8|
      |CHAR;space;$;9|
      |CHAR;quote;$;10|
      |CHAR;escape;$;11|
      (|OutputForm|)
      |CHAR;coerce;$Of;12|
      (|CharacterClass|)
      (10 . |digit|)
      (|Character|)
      (14 . |member?|)
      |CHAR;digit?;$B;13|
```

```

(20 . |hexDigit|)
|CHAR;hexDigit?;$B;14|
(24 . |upperCase|)
|CHAR;upperCase?;$B;15|
(28 . |lowerCase|)
|CHAR;lowerCase?;$B;16|
(32 . |alphanumeric|)
|CHAR;alphanumeric?;$B;17|
(36 . |alphanumeric|)
|CHAR;alphanumeric?;$B;18|
(|String|)
|CHAR;latex;$S;19|
(40 . |minIndex|)
(45 . |elt|)
|CHAR;char;$S;20|
|CHAR;upperCase;2$;21|
|CHAR;lowerCase;2$;22|
(|SingleInteger|))
(QUOTE
#(|^=| 51 |upperCase?| 57 |upperCase| 62 |space| 67
|size| 71 |random| 75 |quote| 79 |ord| 83 |min| 88
|max| 94 |lowerCase?| 100 |lowerCase| 105 |lookup| 110
|latex| 115 |index| 120 |hexDigit?| 125 |hash| 130
|escape| 135 |digit?| 139 |coerce| 144 |char| 149
|alphanumeric?| 159 |alphanumeric?| 164 |>=| 169 |>| 175
|=| 181 |<=| 187 |<| 193))
(QUOTE NIL)
(CONS
(|makeByteWordVec2| 1 (QUOTE (0 0 0 0 0)))
(CONS
(QUOTE #(NIL |OrderedSet&| NIL |SetCategory&|
|BasicType&| NIL))
(CONS
(QUOTE
#((|OrderedFinite|)
(|OrderedSet|)
(|Finite|)
(|SetCategory|)
(|BasicType|)
(|CoercibleTo| 28))))
(|makeByteWordVec2| 52
(QUOTE
(1 8 0 7 9 1 6 0 17 23 0 30 0 31 2 30 12 32 0 33
0 30 0 35 0 30 0 37 0 30 0 39 0 30 0 41 0 30 0
43 1 45 17 0 47 2 45 32 0 17 48 2 0 12 0 0 1 1
0 12 0 38 1 0 0 0 50 0 0 0 25 0 0 15 16 0 0 0 24
0 0 0 26 1 0 17 0 21 2 0 0 0 0 1 2 0 0 0 0 1 1 0
12 0 40 1 0 0 0 51 1 0 19 0 22 1 0 45 0 46 1 0 0
19 20 1 0 12 0 36 1 0 52 0 1 0 0 0 27 1 0 12 0 34
1 0 28 0 29 1 0 0 45 49 1 0 0 17 18 1 0 12 0 44 1
0 12 0 42 2 0 12 0 0 1 2 0 12 0 0 1 2 0 12 0 0 13
2 0 12 0 0 1 2 0 12 0 0 14))))))
(QUOTE |lookupComplete|))

```

Which is a 5 element list. This contains all kinds of information used at runtime by the compiled routines. In particular, functions are looked up at runtime in the first element of the `infovec` list. This first element contains 53 items (in this domain). Item 47 is

```
(40 . |minIndex|)
```

which is the `minIndex` function we seek.

At runtime this `infovec` lives on the property list of the domain name. The domain name of `CHAR` is `Character`. So we look on the property list (a lisp a-list) thus:

```
BOOT>>(symbol-plist '|Character|)

(SYSTEM:DEBUG (:G85875)
 |infovec| (#<vector 08b34380>
           #<vector 08b34364>
           NIL
           (#<bit-vector 08b34310>
            #<vector 08b34348>
            #<vector 08b3432c> . #<vector 08b342f4>)
           |lookupComplete|)
LOADED "/home/axiomgnu/new/mnt/linux/algebra/CHAR.o"
NILADIC T
PNAME "Character"
DATABASE #S(DATABASE
  ABBREVIATION CHAR
  ANCESTORS NIL
  CONSTRUCTOR NIL
  CONSTRUCTORCATEGORY 228064
  CONSTRUCTORKIND |domain|
  CONSTRUCTORMODEMAP 227069
  COSIG (NIL)
  DEFAULTDOMAIN NIL
  MODEMAPS 227404
  NILADIC T
  OBJECT "CHAR"
  OPERATIONALIST 226402
  DOCUMENTATION 152634
  CONSTRUCTORFORM 152626
  ATTRIBUTES 154726
  PREDICATES 154731
  SOURCEFILE "string.spad"
  PARENTS NIL
  USERS NIL
  DEPENDENTS NIL
  SPARE NIL))
```

This list is organized contains many runtime lookup items (notice the `PNAME` entry is “`Character`”, the `LOADED` entry says where the file came from, the `DATABASE` structure entry has database indices (see `daase.lisp.pamphlet` for the structure definition), etc).

Lets get the property list

```
BOOT>>(setq a (symbol-plist '|Character|))

(SYSTEM:DEBUG (:G85875)
 |infovec| (#<vector 08b34380>
```



```

#<vector 08b34364>
NIL
(#<bit-vector 08b34310>
 #<vector 08b34348>
 #<vector 08b3432c> . #<vector 08b342f4>)
|lookupComplete|)
LOADED "/home/axiomgnu/new/mnt/linux/algebra/CHAR.o"
NILADIC T
PNAME "Character"
DATABASE #S(DATABASE
  ABBREVIATION CHAR
  ANCESTORS NIL
  CONSTRUCTOR NIL
  CONSTRUCTORCATEGORY 228064
  CONSTRUCTORKIND |domain|
  CONSTRUCTORMODEMAP 227069
  COSIG (NIL)
  DEFAULTDOMAIN NIL
  MODEMAPS 227404
  NILADIC T
  OBJECT "CHAR"
  OPERATIONALIST 226402
  DOCUMENTATION 152634
  CONSTRUCTORFORM 152626
  ATTRIBUTES 154726
  PREDICATES 154731
  SOURCEFILE "string.spad"
  PARENTS NIL
  USERS NIL
  DEPENDENTS NIL
  SPARE NIL))

```

Next we get the infovec value

```

BOOT>>(setq b (fourth a))

(#<vector 08b34380>
 #<vector 08b34364>
 NIL
 (#<bit-vector 08b34310>
  #<vector 08b34348>
  #<vector 08b3432c> . #<vector 08b342f4>)
|lookupComplete|)

```

Then we get the function table

```

BOOT>>(setq c (car b))

#<vector 08b34380>

```

In this common lisp (GCL) the array is identified by it's memory address.

Notice that it has the right number of entries:

```

BOOT>>(length c)

```

And we can ask for the 47th entry thus:

```
BOOT>>(elt c 47)
```

```
(40 . |minIndex|)
```

Later we end up calling the 48th function (which is elt and returns the actual character in the string). We ask for it:

```
BOOT>>(elt c 48)
```

```
(45 . |elt|)
```

At this point we've reached the metal. Common lisp will evaluate the macro-expanded functions and execute the proper code. Essentially the compiler has changed all of our spad code into runtime table lookups.

## 0.16 The axiom.input startup file

If you add a file in your home directory called ".axiom.input" it will be read and executed when Axiom starts. This is useful for various reasons including setting various switches. Mine reads:

```
)lisp (pprint '‘running /root/.axiom.input’’)
)set quit unprotected
)set message autoload off
)set message startup off
```

You can execute any command in .axiom.input. Be aware that this will ALSO be run while you are doing a "make" so be careful what you ask to do.

## 0.17 Where are Axiom symbols stored?

You'd think that your question about where the symbol is interned would be easy to answer but it is not. The top level loop uses Bill Burge's dreaded zipper parser. You can see it in action by executing the following sequence:

```
)lisp (setq $DALYMODE t)
; this is a special mode of the top level interpreter. If
; $DALYMODE is true then any top-level form that begins
; with an open-paren is considered a lisp expression.
; For almost everything I ever do I end up peeking at the
; lisp so this bit of magic helps.
(trace |intloopProcessString|)
; from int-top.boot.pamphlet
(trace |intloopProcess|)
; the third argument is the "zippered" input
(trace |intloopSpadProcess|)
; now it is all clear, no? sigh.
(trace |phInterpret|)
; from int-top.boot.pamphlet
(trace |intInterpretPform|)
; from intint.lisp.pamphlet
```

```
(trace |processInteractive|)
; from i-toplevel.boot.pamphlet
(setq |$reportInstantiations| t)
; shows what domains were created
(setq |$monitorNewWorld| t)
; watch the interpreter resolve operations
(trace |processInteractive1|)
; from i-toplevel.boot.pamphlet
```

ah HA! I remember now. There is the notion of a “frame” which is basically a namespace in Axiom or an alist in Common Lisp. It is possible to maintain different “frames” and move among them. There is the notion of the current frame and it contains all the defined variables. At any given time the current frame is available as `$InteractiveFrame`. This variable is used in `processInteractive1`. If you do:

```
a:=7
(pprint |$InteractiveFrame|)
```

you’ll see `—a—` show up on the alist. When you do the

```
pgr:=MonoidRing(Polynomial PrimeField 5, Permutation Integer)
p:pgr:=1
```

you’ll see `—p—` show up with 2 other things: (`—p—` mode value) where mode is the “type” of the variable. The value is the internal value. In this case `MonoidRing` has an internal representation. You can find out what the internal representation of a `MonoidRing` is by first asking where the source file is:

```
(do this at a shell prompt, not in axiom)
asq -so MonoidRing ==> mring.spad
```

```
-- or -- in Axiom type:
```

```
)show MonoidRing
```

and you’ll see a line that reads:

```
Issue )edit (yourpath)/../../src/algebra/mring.spad
```

If you look in `mring.spad.pamphlet` you’ll see line 91 that reads:

```
Rep := List Term
```

which says that we will store elements of type `MonoidRing` as a list of `Term` objects. `Term` is defined in the same file (as a macro, which is what `'==>'` means in `spad` files) on line 43:

```
Term ==> Record(coef: R, monom: M)
```

which means that elements of a `MonoidRing` are Lists of Records. The 'R' is defined on line 42 as the first argument to `MonoidRing` which in this case is "Polynomial PrimeField 5". The "M" is also defined on line 42 as the second argument to `MonoidRing` and in this case is "Permutation Integer". So the real representation is

```
List Record(coef: Polynomial PrimeField 5,
            monom: Permutation Integer)
```

In the `$InteractiveFrame` we printed out you can see in the value field that the value is:

```
(|value|
 (|MonoidRing| (|Polynomial| (|PrimeField| 5))
              (|Permutation| (|Integer|)))
 WRAPPED ((0 . 1) . #<vector 08af33d4>))
```

which basically means that we know how the `MonoidRing` was constructed and what its current value is. The `(0 . 1)` likely means that this is the zeroth (constant) term with a leading coefficient of 1. This is just a guess as I haven't decoded the representation of either `Polynomial PrimeField` or `Permutation Integer`. You can do the same deconstruction of these two domains by setting

```
pi:=Permutation Integer
z:pi:=1
```

```
pp5:=Polynomial PrimeField 5
w:pp5:=1
```

and following the same steps as above:

```
(pprint |$InteractiveFrame|)
)show pi
(find the source file)
(find the representation and decode it)
```

```
(pprint |$InteractiveFrame|)
)show pp5
(find the source file)
(find the representation and decode it)
```

Be sure to set `$DALYMODE` to nil if you plan to use Axiom for any real computation. Otherwise every expression that begins with an open-paren will go directly to lisp.

## 0.18 Translating individual boot files to common lisp

If you are making changes to boot code it is sometimes helpful to check the generated lisp code to ensure it does what you want. You can convert an individual boot file to common lisp using the `boottran::boottocl` function:

```
)fin      -- drop into common lisp
(boottran::boottocl "foo.boot")
```

when you do this it creates a foo.clisp file in ../../int/interp

Alternatively if you work from the pamphlet file the process is more painful as you have to do

```
)cd (yourpath)/int/interp
)sys tangle ../../src/interp/foo.boot.pamphlet >foo.boot
)fin
(boottran::boottocl "foo.boot")
(restart)
```

The )cd step tells axiom to cd to the int/interp subdirectory. The )sys tangle... extracts the boot file from the pamphlet file The )fin step drops into common lisp The (boottran... converts the foo.boot file to foo.clisp The (restart) re-enters the top level loop

## 0.19 Directories

For this discussion I assume that you have your system rooted at /spad and was build to run on linux. These directories may not yet be in the CVS tree but are documented here so they make sense when the show up.

The AXIOM variable

The usual setting of the AXIOM variable is /spad/mnt/linux. The name is composed of three parts, the rooted path, in this case /spad, "mnt", and the system you are running, in this case linux. Builds for other systems will have other system names.

/spad

This is the usual root directory of the Axiom system. The name is historical, a contraction of Scratchpad. This name can be anything provided the shell variable AXIOM contains the new prefix.

/spad/mnt

This is a directory which contains files which are specific to a given platform. At a site that contains multiple platforms this directory will contain a subdirectory for each type of platform (e.g. linux, rios, ps2, rt, sun, etc).

/spad/mnt/linux

This directory contains the complete copy of the Axiom system for the linux system. This is the 'mount point' of the system. Executable systems (for RedHat) are shipped relative to this point. In what follows, the ./ refers to /spad/mnt/linux.

\*\*\*\*\*  
There are several directories explained below. They are:

```
./bin      -- user executables
./doc      -- system documentation
./algebra  -- algebra libraries
```

```
./lib    -- system executables
./etc    -- I haven't a clue...
*****
```

## The mnt/linux/bin directory

./bin

This is a directory of user executable commands, either at the top level or thru certain Axiom system calls. Support executables live in ./lib

./bin/htadd

This adds pages to the Hyperdoc database (ht.db, which lives in ./doc/hypertext/pages; hypertext, since we have a penchant for these things, is an historical name for Hyperdoc. The single word 'lawyers' will probably explain away a lot of name changes.)

./bin/spadsys

This is the Axiom interpreter. It is one of the functions started when the user invokes the system using the spadsys command. Normally this command is run under the control of sman (./lib/sman) and the console is under the control of clef (./bin/clef), the wonderful command-line editor. It is possible to start spadsys standalone but it will not talk to Hyperdoc or graphics. Users who rlogin or use an ascii-only terminal (for historical reasons, no doubt) can profit by invoking spadsys directly rather than using ./bin/axiom

./bin/axiom

This is a shell script that spins the world. It kicks off a whole tree of processes necessary to perform the X-related magic we do. It expects the shell variable AXIOM to be set to the 'mount point' (usually to /spad/mnt/linux).

./bin/clef

This is the wonderful command-line editor used by Axiom. It can be used in a stand-alone fashion if you wish.

./bin/SPADEDFN

This script is invoked by the spad )fe command. It can be changed to invoke your favorite editor. While you may invoke your editor, it may not run (as in, yes, I can invoke the devil but will he come when I call?)

./bin/viewalone

This is a function to run the graphics in a stand-alone fashion. The Graphics package (an amazing contribution by several very talented people, most notably Jim Wen and Jon

Steinbach) is a C program that communicates with Axiom thru sockets. It will, however, perform its miracles unaided if invoked by the sufficiently chaste...

`./bin/hypertext`

This is a function to run Hyperdoc (remember the penchant!) stand-alone. The Hyperdoc package owes its existence to the efforts of J.M. Wiley and Scott Morrison. This function works off 'pages' that live in hypertext pages directory and are referenced in the "hyperdoc database" called ht.db (for historical reasons, but you knew that). It is possible for creative plagerists to figure out how to write their own pages and add them to the database (see htadd above), thus gaining fame far and wide...

`./bin/sys-init.lsp`

This is a file of lisp code that gets loaded before Axiom starts. Thus, we distribute patches by adding lisp (load ...) commands to this file. The sufficiently clever should have a field day with this one. (All others should worship the sufficiently clever and send them money, eh?)

`./bin/init.lsp`

This is a file of lisp code loaded if and only if you start spadsys in this directory. The user can put a file of this name in her home directory and it will get loaded at startup with the probable effect of injecting luser errors into the running system. sigh.

## **The mnt/linux/doc directory**

`./doc`

The doc subdirectory contains system documentation.

`./doc/command.list`

This is a file of command completions used by clef when you hit the tab key. This is a little known feature that will surprise someone someday (hopefully pleasantly).

`./doc/book`

This is an attempt at a book describing Axiom. It represents a combination of fantasy, describing what never will be and history (remember the penchant?) describing what was. Any description matching what is may be regarded as failure of the imagination and ignored.

`./doc/compguide`

This is an attempt to describe a compiler that doesn't exist, never did exist, and never will exist. It makes for entertaining reading so we included it.

`./doc/hypertext`

This is the fabled Hyperdoc subdirectory where all of the pages and the database live, along with several other obscure files needed to make the wizards look good.

`./doc/hypertext/pages`

This is where the 'pages' live. Each file ending in `.ht` contains several pages related, if only by chance, to the same topic. You may find it instructive to try to read some of these files. Hyperdoc was learned by the 'campfire' method (sitting around the fire passing along historical facts by word of mouth) and will probably continue to propagate by the same method. Ye may become th' local scribe and soothsayer if ye study the writings here below....

`./doc/hypertext/pages/rootpage.ht`

This file is the magic 'first page' that gets displayed when Hyperdoc starts. There is a macro (see `./doc/hypertext/pages/util.ht`) called `/localinfo` which is intended to allow the luser to add her own pages without modifying the system copies. How this is done was lost when the campfire got rained out.

`./doc/hypertext/pages/util.ht`

This file contains the macros used to extend the system commands. The syntax is hard to learn (it was hard to write, it ought to be hard to learn, eh?).

`./doc/hypertext/pages/ht.db`

This is the Hyperdoc database. It is updated using `./bin/htadd` which must be run whenever a page in this directory gets changed. The necessary arguments to `htadd` are obvious to those in the know.

`./doc/hypertext/bitmaps`

There are several pretty bitmaps used as cursors, buttons and general decorations that hide in this directory.

`./doc/hypertext/ht.files`

This is a list of some Hyperdoc files. It seems to have no purpose in life but it is useful as a koan, as in, What is the length of half a list?

`./doc/hypertext/ht.db`

Another copy of the Hyperdoc database. It isn't clear which one is the real one so I guess we keep both. Maybe we'll figure it out at the friday night campfire provided we don't get too lit.



`./doc/hypertext/gloss.text`

The text used in the glossary. Many magic words lie herein. Some are spoken only by campfire gurus.

`./doc/library`

This is a directory of Hyperdoc pages that can be freely smashed, trashed and generally played with. It uses the `/localinfo` connection to set up a 'library' containing Hyperdoc pages keyed to your favorite textbook. It is interesting to set the shell variable

`HTPATH=/spad/mnt/linux/doc/library:`

`/spad/mnt/linux/doc/hypertext/pages`

and then start Hyperdoc. See the file `./doc/library/macros.ht`

`./doc/msgs`

This directory contains several 'message databases'; the only one of which we seem to care about being `s2-us.msgs` but I can't swear to it.

`./doc/spadhelp`

This is a directory containing help information for a copy of the system that once ran long ago and far away. It is kept for historical reasons (programmers NEVER throw anything away).

`./doc/viewports`

There are several dozen truly fine pictures in Axiom. We have created them and hidden them here. Hyperdoc will insert them at various places (where the text gets too boring, hopefully) and you can click on them there. They get snarfed from here. It is possible to view them with stand-alone graphics but don't ask me how. I missed that campfire due to poisoned marshmallows.

`./doc/complang`

This directory contains fantasy from the past as opposed to facts from the future. Ignore it.

`./doc/ug`

This directory left intentionally blank :- ) (an old IBM joke).

`./doc/tex`

These are the files necessary to create the famous goertler document. If you figure out how to use these please send us the instructions and we will add a log to the campfire with your name on it (a rare honor indeed as luser's names rarely reach the inner circle).

`./doc/htex`

This directory contains the original tex-like source for the luser's guide. There are many functions that munch on these between here and paper but this is approximately where they start. If you do your own algebra perchance you might document it like this. Figuring out the syntax will also get your name into the inner circle (probably connected with a smirk :-) )

./doc/newug

Please don't ask me. I couldn't begin to guess. You wouldn't believe how many 'new' things there are that really aren't. We have more NEW things than Madison Avenue has NEW laundry soap.

./doc/gloss.text

This one is here because it is here. Existentially speaking, of course.

./doc/submitted

This was what the htex files said before history was rewritten... (and renamed?)

## The mnt/linux/algebra directory

./algebra

This is where all of the interesting action lives. Each .NRLIB directory contains 2 files, a code.o and an index.kaf\* file. The code.o contains the executable algebra that gets loaded into the system. The index.kaf\* file contains all kinds of things like signatures, source paths, properties and dried bat droppings. The documentation for each of these can be reached by using the BROWSE feature of Hyperdoc.

./algebra/MODEMAP.daase

This is an inverted database that contains information gleaned from the index.kaf\* files. Without this there is no way to figure out which .NRLIB file to load. This database is opened on startup and kept open.

./algebra/interp.exposed

This is a control file for the interpreter that limits the number of places to search for function names.

\*\*\*\*\*

## The mnt/linux/etc directory

./lib

This directory contains functions that get loaded by the system. Nothing in here is executable by the user but the system needs these functions to run.

./lib/htrefs  
./lib/htsearch  
./lib/hthits

These three functions are used to search the Hyperdoc pages. There is no way in the current system to request a search of those pages so these files are fascinating examples of history in the making...

./lib/hypertext

This is Hyperdoc. What is in a name?

./lib/sman

This is sman, which comes before all. Methinks the name originated as a contraction of superman, the name of a stack frame in a system long ago and far away (VMLisp) chosen because a certain programmer had a penchant for comic books when he was young.

./lib/session  
./lib/spadclient

These two files are processes started by sman for some reason or other. I can never remember what they do or why. However, the campfire fails to smoke if they don't work.

./lib/viewman

This is the controlling function for the graphics.

./lib/view2d

This is invoked when a 2 dimensional window is requested. This is provided mostly for those math majors who never got over the insights from flatland.

./lib/view3d

This is invoked when a 3 dimensional window is requested. Option IBM3634-A is required to convert your 2 dimensional screen to 3 dimensions for realistic viewing. A mathematically accurate, if somewhat more achievable, rendering can be had on a color or monochrome crt without this upgrade.

./lib/gloss.text  
./lib/glosskey.text  
./lib/glossdef.text

These are three files related to the glossary. The first (gloss.text) is the original glossary

text. The second (glosskey.text) is a list of terms and pointers into glossdef.text. The third (glossdef.text for those math majors who can't count) is a list of definitions and pointers back into the second (guess). These files are used by Hyperdoc.

./lib/browsedb.lisp

This is the original file that creates an in-memory hash table used by browse. It is used during system build time. We keep it here to ensure that the bytes on this section of the disk have a well-defined orientation, allowing us to compute the spin vectors of the individual magnetic domains. This allows us to give Heisenburg a sense of direction (at least over the long run).

./lib/comdb.text

./lib/libdb.text

The first file (comdb.text) contains the so-called ++ (plus plus) comments from the algebra files. It contains pointers into the second file. The second file (libdb.text) contains flags (constructor, operation, attribute) and pointers into the first file. These files are used by browse in Hyperdoc.

./lib/loadmprotect

./lib/mprotect

This set of two files has been mercifully de-installed from the system. They will, if used and despite the meaning behind the name, cause random system reboots (yeah, HARDWARE reboots. don't ask me how, I'm just the historian).

./lib/SPAEDIT

./lib/fc

./lib/spadbuf

./lib/SPADEFN

./lib/obey

./lib/ex2ht

I've drawn a blank; intentionally.

## The mnt/linux/lib directory

./etc

This directory intentionally left blank. We just can't figure out WHY we intended to leave it blank. Historical reasons, no doubt.

## 0.20 The )set command

The )set command contains many possible options such as:

## Current Values of )set Variables

| Variable  | Description                                                | Current Value |
|-----------|------------------------------------------------------------|---------------|
| breakmode | execute break processing on error                          | break         |
| compiler  | Library compiler options                                   | ...           |
| expose    | control interpreter constructor exposure                   | ...           |
| functions | some interpreter function options                          | ...           |
| fortran   | view and set options for FORTRAN output                    | ...           |
| kernel    | library functions built into the kernel for efficiency ... |               |
| hyperdoc  | options in using HyperDoc                                  | ...           |
| help      | view and set some help options                             | ...           |
| history   | save workspace values in a history file                    | on            |
| messages  | show messages for various system features                  | ...           |
| naglink   | options for NAGLink                                        | ...           |
| output    | view and set some output options                           | ...           |
| quit      | protected or unprotected quit                              | unprotected   |
| streams   | set some options for working with streams                  | ...           |
| system    | set some system development variables                      | ...           |
| userlevel | operation access level of system user                      | development   |

Variables with current values of ... have further sub-options. For example,  
 issue )set system to see what the options are for system .  
 For more information, issue )help set .

The table that contains these options lives in setvar.boot.pamphlet. The actual code that implements these options is sprinkled around but most of the first-level calls resolve to functions in setvars.boot.pamphlet. Thus if you plan to add a new output style to the system, or figure out where a current style is broken, these two files are the place to start.

A new )set breakmode command has been implemented to handle the case that you might want an error message or an error return code from AXIOMsys. You can set this option with

```
)set breakmode quit
```

This will cause AXIOMsys to exit with the return code of 1. Note that if you invoke the “axiom” shell script to start AXIOMsys you will not see this return code (sman swallows it).

## 0.21 Special Output Formats

The first level of special output formatting is handled by functions in setvar.boot.pamphlet. This handles the options given to the )set command.

## 0.22 Low Level Debugging Techniques

It should be observed that Axiom is basically Common Lisp and some very low level techniques can be used to find where problems occur in algebra code. This section walks thru a small problem and illustrates some techniques that can be used to find bugs. The point of

this exercise is to show a few techniques, not to show a general method.

## Finding Anonymous Function Signatures

This is a technique, adapted from Waldek Hebisch, for asking the interpreter to reveal the actual function that will be called in a given circumstance. Here we have a function `tanint` from the domain `ElementaryIntegration`.

```
tanint(f, x, k) ==
  eta' := differentiate(eta := first argument k, x)
  r1 := tanintegrate(univariate(f, k), differentiate(#1,
    differentiate(#1, x), monomial(eta', 2) + eta'::UP),
    rischDEsys(#1, 2 * eta, #2, #3, x, lflimitedint(#1, x, #2),
      lfextendedint(#1, x, #2)))
  map(multivariate(#1, k), r1.answer) + lfintegrate(r1.a0, x)
```

We would like to know the type signature of the first argument to the inner call to the `differentiate` function:

```
differentiate(#1, x), monomial(eta', 2) + eta'::UP),
```

We see that `differentiate` is called with `#1`, which is Axiom's notation for an anonymous function. How can we determine the signature?

Axiom has a second notation for anonymous functions using the `+->` notation. This notation allows you to explicitly specify type information. In the above code, we would like to replace the `#1` variable with the `+->` and explicit type information.

The first step is to look at the output of the Spad compiler. The abbreviation for `ElementaryIntegration` can be found from the interpreter by:

```
)show ElementaryIntegration
Abbreviation for ElementaryIntegration is INTEF
```

So the compiler output is in the `int/algebra/INTEF.nrlib/code.lsp` file.

There we see the definition of the lisp `tanint` function. Notice that the `$` is a hidden, internal fourth argument to an Axiom three argument function. This is the vector of the current domain containing slots where we can look up information, called the domain vector.

```
(DEFUN |INTEF;tanint| (|f| |x| |k| $)
  (PROG (|eta| |eta'| |r1|)
    (RETURN
      (SEQ
        (LETT |eta'|
          (SPADCALL
            (LETT |eta|
              (|SPADfirst|
                (SPADCALL |k| (QREFELT $ 18)))
                |INTEF;tanint|)
              |x|
              (QREFELT $ 19))
            |INTEF;tanint|)
          (LETT |r1|
            (SPADCALL
              (SPADCALL |f| |k| (QREFELT $ 22))
                (CONS (FUNCTION |INTEF;tanint!1|) (VECTOR |eta'| |x| $))
                (CONS (FUNCTION |INTEF;tanint!4|) (VECTOR |x| $ |eta|)))
```

```

(QREFELT $ 50))
|INTEF;tanint|)
(EXIT
(SPADCALL
(SPADCALL
(CONS
(FUNCTION |INTEF;tanint!5|)
(VECTOR $ |k|))
(QCAR |r1|)
(QREFELT $ 57))
(SPADCALL (QCDR |r1|) |x| (QREFELT $ 58))
(QREFELT $ 59))))))

```

The assignment line for `eta'` is:

```
eta' := differentiate(eta := first argument k, x)
```

which is implemented by the code:

```

(LETT |eta'|
(SPADCALL
(LETT |eta|
(|SPADfirst|
(SPADCALL |k| (QREFELT $ 18)))
|INTEF;tanint|)
|x|
(QREFELT $ 19))
|INTEF;tanint|)

```

from which we see that the inner `differentiate` is slot 19 in the domain vector. Every domain has an associated domain vector which contains references to other functions from other domains, among other things. The `QREFELT` function takes the domain vector `$` and slot number and does a “quick array reference”. The return value is a pair, the car of which is a function to call. The `SPADCALL` macro uses the last argument, in this case the result of `(QREFELT $ 19)` to find the function to call.

The function from slot 19 can be found with:

```

)lisp (setq $dalymode t)
(setf *print-circle* t)
(setf *print-array* nil)
(setf dv (|ElementaryIntegration| (|Integer|) (|Expression| (|Integer|))))
(|replaceGoGetSlot| (cdr (aref dv 19)))
Value = (#<compiled-function |FS-;differentiate;SSS;99|> . #<vector 090cbccc>)

```

The call of `(setq $dalymode t)` changes the Axiom top level loop to interpret any input that begins with an open parenthesis to be interpreted as a lisp s-expression rather than Axiom input. This saves typing `)lisp` in front of every lisp expression. Be sure to do a `(setq $dalymode nil)` when you are finished.

The `*print-circle*` needs to be true because the domain vector contains circular references to itself and we need to make sure that we check for this during printing so the print is not infinite.

The `*print-array*` needs to be nil so that the arrays just print some identifying information rather than the detailed array contents.

The `(setf dv ...)` uses the Lisp internal names for the domains. In Axiom, the names of types are case-sensitive symbols. These are represented in lisp surrounded by vertical bars

because lisp is not case sensitive. The dv variable is essentially being set to the Axiom equivalent of:

```
dv:=ElementaryIntegration(Integer,Expression(Integer))
```

except we do this in lisp. The end result is that dv will contain the domain vector for the newly constructed domain. From the lisp code

Consider the call of the form:

```
(SPADCALL A B '(C . D))
```

The SPADCALL macro takes a set of arguments, the last of which is a pair where C is the function to call and D is the domain vector. So if we do:

```
(macroexpand-1 '(spadcall a b '(c . d)))
```

Value =

```
(LET ((#0=#:G1417 (QUOTE (C . D))))
  (THE (VALUES T) (FUNCALL (CAR #0#) A B (CDR #0#))))
```

Note that #0 is a “pointer”, in this case to the list '(c) and #0# is a use of that pointer. This is done to make sure that you reference the exact cons cell of the argument.

In Axiom compiler output

```
(SPADCALL eta k (QREFELT $ 19))
```

approximately translates to

```
(FUNCALL (CAR (QREFELT $ 19)) eta k (CDR (QREFELT $ 19)))
```

which calls the function from the domain slot 19 on the value assigned to eta and the variable k and the domain. Thus, the full expansion becomes

```
(FUNCALL #<compiled-function |FS-;differentiate;SSS;99|>
  eta k #<vector 090cbccc>)
```

From this we can see a reference to FS-;differentiate;SSS;99 which is the internal name of the differentiate function from the FS- category.

Note that FunctionSpace is a category. When categories contain implementation code the compiler generates 2 nrlibs. The Axiom convention for categorical implementation of code using a trailing “-” so the actual code for FS-;differentiate;SSS;99 lives in int/algebra/FS-.nrlib/code.lsp

We can see that the differentiate function is coming from the category

```
)show FS
```

```
FunctionSpace R: OrderedSet is a category constructor
```

```
Abbreviation for FunctionSpace is FS
```

```
....
```

```
differentiate : (%,Symbol) -> % if R has RING
```

```
differentiate : (%,List Symbol) -> % if R has RING
```

```
differentiate : (%,Symbol,NonNegativeInteger) -> % if R has RING
```

```
differentiate : (%,List Symbol,List NonNegativeInteger) -> % if R has RING
```

From the above signatures we know there is only one differentiate that is a two argument form so the call

```
differentiate(#1, x), monomial(eta', 2) + eta'::UP),
```

must be the first instance.



From the sources (bookvol10.4) we see that the `tanint` function has the signature:

```
tanint      : (F, SE, K) -> IR
```

and that

```
SE ==> Symbol
F  : Join(AlgebraicallyClosedField, TranscendentalFunctionCategory,
         FunctionSpace R)
K  ==> Kernel F
```

The `differentiate` function takes something of type `F` and a `Symbol` and returns something of type `F`. If we write this as an anonymous function it becomes:

```
(x2 : F) : F +-> differentiate(x2, x)
```

Thus, we can rewrite the `differentiate` call as:

```
differentiate(#1, x), monomial(eta', 2) + eta'::UP),
```

as

```
(x2 : F) : F +-> differentiate(x2, x),
monomial(eta', 2) + eta'::UP),
```

Continuing in this way we can fully rewrite the assignments as:

```
r1 := tanintegrate(univariate(f, k),
  (x1 : UP) : UP +-> differentiate(x1,
    (x2 : F) : F +-> differentiate(x2, x),
    monomial(eta', 2) + eta'::UP),
  (x6 : Integer, x2 : F, x3 : F) : Union(List F, "failed") +->
    rischDEsys(x6, 2 * eta, x2, x3, x,
      (x4 : F, x5 : List F) : U3 +-> lflimitedint(x4, x, x5),
      (x4 : F, x5 : F) : U2 +-> lfextendedint(x4, x, x5)))
map((x1 : RF) : F +-> multivariate(x1, k), r1.answer) + _
  lfintegrate(r1.a0, x)
```

Note that `rischDEsys` is tricky, because `rischDEsys` returns only `List F`, but `tanintegrate` expects union.

## The example bug

Axiom can generate TeX output by typing:

```
)set output tex on
```

Here we give an example of TeX output that contains a bug:

```
(1) -> )set output tex on
(1) -> radix(10**10,32)
Loading /axiom/mnt/linux/algebra/RADUTIL.o for package
  RadixUtilities
Loading /axiom/mnt/linux/algebra/RADIX.o
  for domain RadixExpansion
Loading /axiom/mnt/linux/algebra/ANY1.o
  for package AnyFunctions1
Loading /axiom/mnt/linux/algebra/NONE1.o
  for package NoneFunctions1
Loading /axiom/mnt/linux/algebra/ANY.o
  for domain Any
```

```

Loading /axiom/mnt/linux/algebra/SEX.o
  for domain SExpression

(1) 9AONP00
Loading /axiom/mnt/linux/algebra/TEX.o
  for domain TexFormat
Loading /axiom/mnt/linux/algebra/CCLASS.o
  for domain CharacterClass
Loading /axiom/mnt/linux/algebra/IBITS.o
  for domain IndexedBits
Loading /axiom/mnt/linux/algebra/UNISEG.o
  for domain UniversalSegment
$$
9#\A0#\N#\P00
\leqno(1)
$$
  Loading /axiom/mnt/linux/algebra/VOID.o for domain Void

                                         Type: RadixExpansion 32

```

The correct output should be:

```

$$
9AONP00
\leqno(1)
$$

```

So we need to figure out where the # prefixes are being generated. In the above code we can see various domains being loaded. These domains are lisp code. Each domain lives in a subdirectory of its own. For example, the ANY domain lives in ANY.NRLIB. The ANY.NRLIB directory contains a common lisp file named code.lsp. The compiled form of this code ANY.o is loaded whenever the domain Any is referenced. We can look at the lisp code:

```

(/VERSIONCHECK 2)

(PUT (QUOTE |ANY;obj;$N;1|)
      (QUOTE |SPADreplace|)
      (QUOTE QCDR))

(DEFUN |ANY;obj;$N;1| (|x| $) (QCDR |x|))

(PUT (QUOTE |ANY;dom;$Se;2|)
      (QUOTE |SPADreplace|)
      (QUOTE QCAR))

(DEFUN |ANY;dom;$Se;2| (|x| $) (QCAR |x|))

(PUT (QUOTE |ANY;domainOf;$Of;3|)
      (QUOTE |SPADreplace|)
      (QUOTE QCAR))

(DEFUN |ANY;domainOf;$Of;3| (|x| $) (QCAR |x|))

(DEFUN |ANY;=;2$B;4| (|x| |y| $)

```

```

(COND
  ((SPADCALL (QCAR |x|) (QCAR |y|) (QREFELT $ 17))
   (EQ (QCDR |x|) (QCDR |y|)))
  ((QUOTE T) (QUOTE NIL))))

(DEFUN |ANY;objectOf;$Of;5| (|x| $)
  (|spad2BootCoerce|
   (QCDR |x|)
   (QCAR |x|)
   (SPADCALL
    (SPADCALL "OutputForm" (QREFELT $ 21))
    (QREFELT $ 23))))

(DEFUN |ANY;showTypeInOutPut;BS;6| (|b| $)
  (SEQ
   (SETELT $ 10 (SPADCALL |b| (QREFELT $ 9)))
   (EXIT
    (COND
     (|b| "Type of object will be displayed in
          output of a member of Any")
     ((QUOTE T) "Type of object will not be displayed in
                output of a member of Any")))))

(DEFUN |ANY;coerce;$Of;7| (|x| $)
  (PROG (|obj1| |p| |dom1| #0#:#:G1426 |a| #1#:#:G1427)
  (RETURN
   (SEQ
    (LETT |obj1|
          (SPADCALL |x| (QREFELT $ 24))
          |ANY;coerce;$Of;7|)
    (COND
     ((NULL (SPADCALL (QREFELT $ 10) (QREFELT $ 26)))
      (EXIT |obj1|)))
    (LETT |dom1|
          (SEQ
           (LETT |p|
                (|prefix2String| (|devaluate| (QCAR |x|)))
                |ANY;coerce;$Of;7|)
           (EXIT
            (COND
             ((SPADCALL |p| (QREFELT $ 27))
              (SPADCALL |p| (QREFELT $ 23)))
             ((QUOTE T) (SPADCALL |p| (QREFELT $ 29))))))
          |ANY;coerce;$Of;7|)
    (EXIT
     (SPADCALL
      (CONS |obj1|
            (CONS ":"
                  (PROGN
                   (LETT #0# NIL |ANY;coerce;$Of;7|)
                   (SEQ
                    (LETT |a| NIL |ANY;coerce;$Of;7|)
                    (LETT #1# |dom1| |ANY;coerce;$Of;7|)
                    G190

```

```

(COND
  ((OR (ATOM #1#)
        (PROGN
          (LETT |a| (CAR #1#) |ANY;coerce;$Of;7|)
          NIL))
        (GO G191)))
  (SEQ
    (EXIT
      (LETT #0#
        (CONS
          (SPADCALL |a| (QREFELT $ 30))
          #0#)
          |ANY;coerce;$Of;7|)))
    (LETT #1# (CDR #1#) |ANY;coerce;$Of;7|)
    (GO G190)
    G191
    (EXIT (NREVERSEO #0#))))))
(QREFELT $ 31))))))

(DEFUN |ANY;any;SeN$;8| (|domain| |object| $)
  (SEQ
    (COND
      ((|isValidType| |domain|) (CONS |domain| |object|))
      ((QUOTE T)
        (SEQ
          (LETT |domain| (|devaluate| |domain|) |ANY;any;SeN$;8|)
          (EXIT
            (COND
              ((|isValidType| |domain|) (CONS |domain| |object|))
              ((QUOTE T)
                (|error|
                  "function any must have a domain as first argument"))))))))

(DEFUN |Any| NIL
  (PROG NIL
    (RETURN
      (PROG (#0=#:G1432)
        (RETURN
          (COND
            ((LETT #0#
              (HGET |$ConstructorCache| (QUOTE |Any|))
              |Any|)
              (|CDRwithIncrement| (CDAR #0#)))
            ((QUOTE T)
              (UNWIND-PROTECT
                (PROG1
                  (CDDAR
                    (HPUT |$ConstructorCache|
                      (QUOTE |Any|)
                      (LIST (CONS NIL (CONS 1 (|Any|))))))
                  (LETT #0# T |Any|))
                (COND
                  ((NOT #0#)
                    (HREM |$ConstructorCache| (QUOTE |Any|))))))))))

```

```

(DEFUN |Any| |NIL
  (PROG (|dv$| $ |pv$|)
    (RETURN
      (PROGN
        (LETT |dv$| (QUOTE (|Any|)) . #0=(|Any|))
        (LETT $ (make-array 35) . #0#)
        (QSETREFV $ 0 |dv$|)
        (QSETREFV $ 3 (LETT |pv$| (|buildPredVector| 0 0 NIL) . #0#))
        (|haddProp| |$ConstructorCache| (QUOTE |Any|) NIL (CONS 1 $))
        (|stuffDomainSlots| $)
        (QSETREFV $ 6
          (|Record| (|:| |dm| (|SExpression|)) (|:| |obj| (|None|))))
        (QSETREFV $ 10 (SPADCALL (QUOTE NIL) (QREFELT $ 9)))
        $))))

(setf (get (QUOTE |Any|) (QUOTE |infovec|))
  (LIST
    (QUOTE
      #(NIL NIL NIL NIL NIL NIL (QUOTE |Rep|)
        (|Boolean|) (|Reference| 7) (0 . |ref|)
        (QUOTE |printTypeInOutPut|) (|None|)
        |ANY;obj;$N;1| (|SExpression|) |ANY;dom;$Se;2|
        (|OutputForm|) |ANY;domainOf;$Of;3| (5 . =)
        |ANY;=;2$B;4| (|String|) (|Symbol|) (11 . |coerce|)
        (|List| 20) (16 . |list|) |ANY;objectOf;$Of;5|
        |ANY;showTypeInOutPut;BS;6| (21 . |deref|)
        (26 . |atom?|) (|List| $) (31 . |list|)
        (36 . |coerce|) (41 . |hconcat|) |ANY;coerce;$Of;7|
        |ANY;any;SeN$;8| (|SingleInteger|)))
    (QUOTE #(~= 46 |showTypeInOutPut| 52 |objectOf| 57 |obj|
      62 |latex| 67 |hash| 72 |domainOf| 77 |dom| 82
      |coerce| 87 |any| 92 = 98))
    (QUOTE NIL)
    (CONS (|makeByteWordVec2| 1 (QUOTE (0 0 0)))
      (CONS (QUOTE #(|SetCategory&| |BasicType&| NIL))
        (CONS
          (QUOTE #((|SetCategory|) (|BasicType|) (|CoercibleTo| 15)))
            (|makeByteWordVec2| 34
              (QUOTE (1 8 0 7 9 2 13 7 0 0 17 1 20 0 19 21 1 22 0 20
                23 1 8 7 0 26 1 13 7 0 27 1 20 28 0 29 1 20 15
                0 30 1 15 0 28 31 2 0 7 0 0 1 1 0 19 7 25 1 0
                15 0 24 1 0 11 0 12 1 0 19 0 1 1 0 34 0 1 1 0
                15 0 16 1 0 13 0 14 1 0 15 0 32 2 0 0 13 11 33
                2 0 7 0 0 18))))))
    (QUOTE |lookupComplete|)))

(setf (get (QUOTE |Any|) (QUOTE NILADIC)) T)

```

We can ignore this information and focus on the functions that are defined in this file. These functions can be traced with the usual common lisp tracing facility. So lets create a file /tmp/debug.lisp that contains a trace statement for each DEFUN in ANY.NRLIB/code.lsp. It looks like:

```
(trace |ANY1;retractable?;AB;1|)
```

```
(trace |ANY1;coerce;SA;2|)
(trace |ANY1;retractIfCan;AU;3|)
(trace |ANY1;retract;AS;4|)
(trace |AnyFunctions1|)
(trace |AnyFunctions1;|)
```

We can now restart the axiom system, rerun the failing expression (this will autoload ANY.o; alternatively we could hand-load the ANY.NRLIB/code.lsp file), and then load our /tmp/debug.lisp file. Now all of the functions in the ANY domain are traced and we can watch the trace occur while the expression is evaluated. In this example I've created a larger file that traces all of the loaded domains:

```
(trace |RADUTIL;radix;FIA;1|)
(trace |RadixUtilities|)
(trace |RadixUtilities;|)

(trace |RADIX;characteristic;Mni;1|)
(trace |RADIX;differentiate;2$;2|)
(trace |RADIX;Zero;$;3|)
(trace |RADIX;One;$;4|)
(trace |RADIX;-;2$;5|)
(trace |RADIX;+;3$;6|)
(trace |RADIX;-;3$;7|)
(trace |RADIX;*;I2$;8|)
(trace |RADIX;*;3$;9|)
(trace |RADIX;/;3$;10|)
(trace |RADIX;/;2I$;11|)
(trace |RADIX;<;2$B;12|)
(trace |RADIX;=;2$B;13|)
(trace |RADIX;numer;$I;14|)
(trace |RADIX;denom;$I;15|)
(trace |RADIX;coerce;$F;16|)
(trace |RADIX;coerce;I$;17|)
(trace |RADIX;coerce;F$;18|)
(trace |RADIX;retractIfCan;$U;19|)
(trace |RADIX;retractIfCan;$U;20|)
(trace |RADIX;ceiling;$I;21|)
(trace |RADIX;floor;$I;22|)
(trace |RADIX;wholePart;$I;23|)
(trace |RADIX;fractionPart;$F;24|)
(trace |RADIX;wholeRagits;$L;25|)
(trace |RADIX;fractRagits;$S;26|)
(trace |RADIX;prefixRagits;$L;27|)
(trace |RADIX;cycleRagits;$L;28|)
(trace |RADIX;wholeRadix;L$;29|)
(trace |RADIX;fractRadix;2L$;30|)
(trace |RADIX;intToExpr|)
(trace |RADIX;exprgroup|)
(trace |RADIX;intgroup|)
(trace |RADIX;overBar|)
(trace |RADIX;coerce;$Of;35|)
(trace |RADIX;checkRagits|)
(trace |RADIX;radixInt|)
(trace |RADIX;radixFrac|)
(trace |RadixExpansion|)
```

```

(trace |RadixExpansion;|)

(trace |ANY1;retractable?;AB;1|)
(trace |ANY1;coerce;SA;2|)
(trace |ANY1;retractIfCan;AU;3|)
(trace |ANY1;retract;AS;4|)
(trace |AnyFunctions1|)
(trace |AnyFunctions1;|)

(trace |NONE1;coerce;SN;1|)
(trace |NoneFunctions1|)
(trace |NoneFunctions1;|)

(trace |ANY;obj;$N;1|)
(trace |ANY;dom;$Se;2|)
(trace |ANY;domainOf;$Of;3|)
(trace |ANY;=;2$B;4|)
(trace |ANY;objectOf;$Of;5|)
(trace |ANY;showTypeInOutput;BS;6|)
(trace |ANY;coerce;$Of;7|)
(trace |ANY;any;SeN$;8|)
(trace |Any|)
(trace |Any;|)

(trace |SExpression|)
(trace |SExpression;|)

(trace |TEX;new;$;1|)
(trace |TEX;newWithNum|)
(trace |TEX;coerce;Of$;3|)
(trace |TEX;convert;OfI$;4|)
(trace |TEX;display;$IV;5|)
(trace |TEX;display;$V;6|)
(trace |TEX;prologue;$L;7|)
(trace |TEX;tex;$L;8|)
(trace |TEX;epilogue;$L;9|)
(trace |TEX;setPrologue!;$2L;10|)
(trace |TEX;setTex!;$2L;11|)
(trace |TEX;setEpilogue!;$2L;12|)
(trace |TEX;coerce;$Of;13|)
(trace |TEX;ungroup|)
(trace |TEX;postcondition|)
(trace |TEX;stringify|)
(trace |TEX;lineConcat|)
(trace |TEX;splitLong|)
(trace |TEX;splitLong1|)
(trace |TEX;group|)
(trace |TEX;addBraces|)
(trace |TEX;addBrackets|)
(trace |TEX;parenthesize|)
(trace |TEX;precondition|)
(trace |TEX;formatSpecial|)
(trace |TEX;formatPlex|)
(trace |TEX;formatMatrix|)

```

```
(trace |TEX;formatFunction|)
(trace |TEX;formatNullary|)
(trace |TEX;formatUnary|)
(trace |TEX;formatBinary|)
(trace |TEX;formatNary|)
(trace |TEX;formatNaryNoGroup|)
(trace |TEX;formatTex|)
(trace |TexFormat|)
(trace |TexFormat;|)
```

```
(trace |CCLASS;digit;$;1|)
(trace |CCLASS;hexDigit;$;2|)
(trace |CCLASS;upperCase;$;3|)
(trace |CCLASS;lowerCase;$;4|)
(trace |CCLASS;alphabetic;$;5|)
(trace |CCLASS;alphanumeric;$;6|)
(trace |CCLASS;=;2$B;7|)
(trace |CCLASS;member?;C$B;8|)
(trace |CCLASS;union;3$;9|)
(trace |CCLASS;intersect;3$;10|)
(trace |CCLASS;difference;3$;11|)
(trace |CCLASS;complement;2$;12|)
(trace |CCLASS;convert;$S;13|)
(trace |CCLASS;convert;$L;14|)
(trace |CCLASS;charClass;$S;15|)
(trace |CCLASS;charClass;$L;16|)
(trace |CCLASS;coerce;$Of;17|)
(trace |CCLASS;#;$Nni;18|)
(trace |CCLASS;empty;$;19|)
(trace |CCLASS;brace;$;20|)
(trace |CCLASS;insert!;C2$;21|)
(trace |CCLASS;remove!;C2$;22|)
(trace |CCLASS;inspect;$C;23|)
(trace |CCLASS;extract!;$C;24|)
(trace |CCLASS;map;M2$;25|)
(trace |CCLASS;map!;M2$;26|)
(trace |CCLASS;parts;$L;27|)
(trace |CharacterClass|)
(trace |CharacterClass;|)
```

```
(trace |IBITS;minIndex;$I;1|)
(trace |IBITS;range|)
(trace |IBITS;coerce;$Of;3|)
(trace |IBITS;new;NniB$;4|)
(trace |IBITS;empty;$;5|)
(trace |IBITS;copy;2$;6|)
(trace |IBITS;#;$Nni;7|)
(trace |IBITS;=;2$B;8|)
(trace |IBITS;<;2$B;9|)
(trace |IBITS;and;3$;10|)
(trace |IBITS;or;3$;11|)
(trace |IBITS;xor;3$;12|)
(trace |IBITS;setelt;$I2B;13|)
(trace |IBITS;elt;$IB;14|)
```



```

(trace |IBITS;Not;2$;15|)
(trace |IBITS;And;3$;16|)
(trace |IBITS;Or;3$;17|)
(trace |IndexedBits|)
(trace |IndexedBits;|)

(trace |UNISEG;segment;$S;1|)
(trace |UNISEG;segment;2S$;2|)
(trace |UNISEG;BY;$I$;3|)
(trace |UNISEG;lo;$S;4|)
(trace |UNISEG;low;$S;5|)
(trace |UNISEG;hasHi;$B;6|)
(trace |UNISEG;hi;$S;7|)
(trace |UNISEG;high;$S;8|)
(trace |UNISEG;incr;$I;9|)
(trace |UNISEG;SEGMENT;$S;10|)
(trace |UNISEG;SEGMENT;2S$;11|)
(trace |UNISEG;coerce;$S;12|)
(trace |UNISEG;convert;$S;13|)
(trace |UNISEG;=;2$B;14|)
(trace |UNISEG;coerce;$Of;15|)
(trace |UNISEG;expand;$S;16|)
(trace |UNISEG;map;M$S;17|)
(trace |UNISEG;plusInc|)
(trace |UNISEG;expand;LS;19|)
(trace |UNISEG;expand;LS;19!0|)
(trace |UniversalSegment|)
(trace |UniversalSegment;|)

```

Now we rerun the function and get the trace output

```
(2) -> )lisp (load "/axiom/debug.lisp")
```

Value = T

```

(2) -> radix(10**10,32)

1> (|RadixUtilities|)
<1 (|RadixUtilities| #<vector 08b565cc>)
1> (|RadixExpansion| 32)
<1 (|RadixExpansion| #<vector 08b8cc94>)
1> (|AnyFunctions1| #<vector 08b8cc94>)
<1 (|AnyFunctions1| #<vector 08b5647c>)
1> (|RadixExpansion| 32)
<1 (|RadixExpansion| #<vector 08b8cc94>)
1> (|RADIX;radixInt| 10000000000 32 #<vector 08b8cc94>)
<1 (|RADIX;radixInt| (9 10 0 23 25 0 0))
1> (|RADIX;radixFrac| 0 1 32 #<vector 08b8cc94>)
<1 (|RADIX;radixFrac| (NIL 0))

1> (|RadixExpansion| 32)
<1 (|RadixExpansion| #<vector 08b8cc94>)
1> (|RADIX;intgroup| (9 10 0 23 25 0 0) #<vector 08b8cc94>)
2> (|RADIX;intToExpr| 9 #<vector 08b8cc94>)
<2 (|RADIX;intToExpr| 9)

```

```

2> (|RADIX;intToExpr| 10 #<vector 08b8cc94>)
<2 (|RADIX;intToExpr| #\A)
2> (|RADIX;intToExpr| 0 #<vector 08b8cc94>)
<2 (|RADIX;intToExpr| 0)
2> (|RADIX;intToExpr| 23 #<vector 08b8cc94>)
<2 (|RADIX;intToExpr| #\N)
2> (|RADIX;intToExpr| 25 #<vector 08b8cc94>)
<2 (|RADIX;intToExpr| #\P)
2> (|RADIX;intToExpr| 0 #<vector 08b8cc94>)
<2 (|RADIX;intToExpr| 0)
2> (|RADIX;intToExpr| 0 #<vector 08b8cc94>)
<2 (|RADIX;intToExpr| 0)
<1 (|RADIX;intgroup| (CONCAT 9 #\A 0 #\N #\P 0 0))
1> (|RADIX;exprgroup|
  ((CONCAT 9 #\A 0 #\N #\P 0 0)) #<vector 08b8cc94>)
<1 (|RADIX;exprgroup| (CONCAT 9 #\A 0 #\N #\P 0 0))
  (2) 9AONPOO
1> (|TeXFormat|)
<1 (|TeXFormat| #<vector 08b24000>)
1> (|TeXFormat|)
<1 (|TeXFormat| #<vector 08b24000>)
1> (|TEX;newWithNum| 2 #<vector 08b24000>)
<1 (|TEX;newWithNum| #<vector 08b8c284>)
1> (|TEX;precondition|
  (CONCAT 9 #\A 0 #\N #\P 0 0) #<vector 08b24000>)
<1 (|TEX;precondition| (CONCAT 9 #\A 0 #\N #\P 0 0))
1> (|TEX;formatTex|
  (CONCAT 9 #\A 0 #\N #\P 0 0) 0 #<vector 08b24000>)
2> (|TEX;stringify| CONCAT #<vector 08b24000>)
<2 (|TEX;stringify| "CONCAT")
2> (|TEX;formatSpecial| "CONCAT"
  (9 #\A 0 #\N #\P 0 0) 0 #<vector 08b24000>)
3> (|TEX;formatNary| ""
  (9 #\A 0 #\N #\P 0 0) 0 #<vector 08b24000>)
4> (|TEX;formatNaryNoGroup| ""
  (9 #\A 0 #\N #\P 0 0) 0 #<vector 08b24000>)
5> (|TEX;formatTex| 9 0 #<vector 08b24000>)
6> (|TEX;stringify| 9 #<vector 08b24000>)
<6 (|TEX;stringify| "9")
<5 (|TEX;formatTex| "9")
5> (|TEX;formatTex| #\A 0 #<vector 08b24000>)
6> (|TEX;stringify| #\A #<vector 08b24000>)
<6 (|TEX;stringify| "#\A")
6> (|IBITS;range|
  #<bit-vector 0831d930> 35 #<vector 085da658>)
<6 (|IBITS;range| 35)
<5 (|TEX;formatTex| "#\A")
5> (|TEX;formatTex| 0 0 #<vector 08b24000>)
6> (|TEX;stringify| 0 #<vector 08b24000>)
<6 (|TEX;stringify| "0")
<5 (|TEX;formatTex| "0")
5> (|TEX;formatTex| #\N 0 #<vector 08b24000>)
6> (|TEX;stringify| #\N #<vector 08b24000>)
<6 (|TEX;stringify| "#\N")

```

```

6> (|IBITS;range|
    #<bit-vector 0831d930> 35 #<vector 085da658>)
<6 (|IBITS;range| 35)
<5 (|TEX;formatTex| "#\N")
5> (|TEX;formatTex| #\P 0 #<vector 08b24000>)
6> (|TEX;stringify| #\P #<vector 08b24000>)
<6 (|TEX;stringify| "#\P")
6> (|IBITS;range|
    #<bit-vector 0831d930> 35 #<vector 085da658>)
<6 (|IBITS;range| 35)
<5 (|TEX;formatTex| "#\P")
5> (|TEX;formatTex| 0 0 #<vector 08b24000>)
6> (|TEX;stringify| 0 #<vector 08b24000>)
<6 (|TEX;stringify| "0")
<5 (|TEX;formatTex| "0")
5> (|TEX;formatTex| 0 0 #<vector 08b24000>)
6> (|TEX;stringify| 0 #<vector 08b24000>)
<6 (|TEX;stringify| "0")
<5 (|TEX;formatTex| "0")
<4 (|TEX;formatNaryNoGroup| "9#\A0#\N#\P00")
4> (|TEX;group| "9#\A0#\N#\P00" #<vector 08b24000>)
<4 (|TEX;group| "{9#\A0#\N#\P00}")
<3 (|TEX;formatNary| "{9#\A0#\N#\P00}")
<2 (|TEX;formatSpecial| "{9#\A0#\N#\P00}")
<1 (|TEX;formatTex| "{9#\A0#\N#\P00}")
1> (|TEX;postcondition|
    "{9#\A0#\N#\P00}" #<vector 08b24000>)
2> (|TEX;ungroup| "{9#\A0#\N#\P00}" #<vector 08b24000>)
<2 (|TEX;ungroup| "9#\A0#\N#\P00")
<1 (|TEX;postcondition| "9#\A0#\N#\P00")
$$
1> (|TEX;splitLong|
    "9#\A0#\N#\P00" 77 #<vector 08b24000>)
2> (|TEX;splitLong|
    "9#\A0#\N#\P00" 77 #<vector 08b24000>)
3> (|TEX;lineConcat|
    "9#\A0#\N#\P00 " NIL #<vector 08b24000>)
<3 (|TEX;lineConcat| ("9#\A0#\N#\P00 "))
<2 (|TEX;splitLong| ("9#\A0#\N#\P00 "))
<1 (|TEX;splitLong| ("9#\A0#\N#\P00 "))
9#\A0#\N#\P00
\leqno(2)
$$

```

Type: RadixExpansion 32

Notice the call that reads:

```

2> (|RADIX;intToExpr| 10 #<vector 08b8cc94>)
<2 (|RADIX;intToExpr| #\A)

```

This means that calling —RADIX;intToExpr— with the number 10 and “the domain vector” generates the character #

A which fails. If we had the domain vector in a variable we could hand-execute this algebra function directly and watch it fail. So we go to the file RADIX.NRLIB/code.lsp which

contains the definition of `RADIX;intToExpr`. The definition is:

```
(DEFUN |RADIX;intToExpr| (|i| $)
  (COND
    ((< |i| 10)
      (SPADCALL |i| (QREFELT $ 66)))
    ((QUOTE T)
      (SPADCALL
        (SPADCALL
          (QREFELT $ 64)
          (+ (- |i| 10) (SPADCALL (QREFELT $ 64) (QREFELT $ 68))))
          (QREFELT $ 70))
        (QREFELT $ 71))))))
```

We can put this definition into our `/tmp/debug.lisp` file and modify it to capture the domain vector passed in the `$` variable thus:

```
(DEFUN |RADIX;intToExpr| (|i| $)
  (setq tpd $)
  (COND
    ((< |i| 10)
      (SPADCALL |i| (QREFELT $ 66)))
    ((QUOTE T)
      (SPADCALL
        (SPADCALL
          (QREFELT $ 64)
          (+ (- |i| 10) (SPADCALL (QREFELT $ 64) (QREFELT $ 68))))
          (QREFELT $ 70))
        (QREFELT $ 71))))))
```

Now when this function is executed the `tpd` variable will contain the value of `$`, the domain vector. So we load `debug.lisp` again to redefine `RADIX;intToExpr` and re-execute the function. The trace results will be the same but now the global variable `tpd` will have the domain vector:

```
(4) -> (identity tpd)
```

```
Value = #<vector 08b8cc94>
```

Now we can use common lisp to step the `RADIX;intToExpr` function:

```
(4) -> (step (|RADIX;intToExpr| 10 tpd))
```

Type `?` and a newline for help.

```
(|RADIX;intToExpr| 10 ...) ?
```

Stepper commands:

```
n (or N or Newline): advances to the next form.
s (or S):           skips the form.
p (or P):           pretty-prints the form.
f (or F) FUNCTION: skips until the FUNCTION is called.
q (or Q):           quits.
u (or U):           goes up to the enclosing form.
e (or E) FORM:      evaluates the FORM and prints the value(s).
r (or R) FORM:      evaluates the FORM and returns the value(s).
b (or B):           prints backtrace.
?:                 prints this.
```

```

(|RADIX;intToExpr| 10 ...)
10
TPD
  = #<vector 08b8cc94>
    (SYSTEM::TRACE-CALL (QUOTE #:G1624) ...)
(QQUOTE #:G1624)
SYSTEM::ARGS
  = (10 #<vector 08b8cc94>)
    (QUOTE T)
(QQUOTE T)
(QQUOTE (CONS # ...))
(QQUOTE T)
(QQUOTE (CONS # ...))
(LET (#) ...)
(QQUOTE (10 #<vector 08b8cc94>))
T
  = T
  1> (LET (#) ...)
(QQUOTE (10 #<vector 08b8cc94>))
(CONS (QUOTE |RADIX;intToExpr|) ...)
(QQUOTE |RADIX;intToExpr|)
SYSTEM::ARGLIST
  = (10 #<vector 08b8cc94>)
    = (|RADIX;intToExpr| 10 ...)
    = (|RADIX;intToExpr| 10 ...)
(|RADIX;intToExpr| 10 ...)
  (SETQ TPD ...)
$
  = #<vector 08b8cc94>
  = #<vector 08b8cc94>
  (COND (# #) ...)
(< |i| ...)
|i|
  = 10
  10
  = NIL
  (QUOTE T)
(SPADCALL (SPADCALL # ...) ...)
(LET (#) ...)
(QREFELT $ ...)
(SVREF $ ...)
$
  = #<vector 08b8cc94>
  71
  = (#<compiled-function |CHAR;coerce;$0f;12|> .
    #<vector 08b3901c>)
  = (#<compiled-function |CHAR;coerce;$0f;12|> .
    #<vector 08b3901c>)
    (THE (VALUES T) ...)
(FUNCALL (CAR #:G1776) ...)
(CAR #:G1776)
#:G1776
  = (#<compiled-function |CHAR;coerce;$0f;12|> .
    #<vector 08b3901c>)

```

```

= #<compiled-function |CHAR;coerce;$0f;12|>
(SPADCALL (QREFELT $ ...) ...)

(LET (#) ...)
(QREFELT $ ...)
(SVREF $ ...)
$
= #<vector 08b8cc94>
70
= (#<compiled-function
  |ISTRING;elt;$IC;30|> .
  #<vector 08b26850>)
= (#<compiled-function
  |ISTRING;elt;$IC;30|> .
  #<vector 08b26850>)
(THE (VALUES T) ...)
(FUNCALL (CAR #:G1777) ...)
(CAR #:G1777)
#:G1777
= (#<compiled-function
  |ISTRING;elt;$IC;30|> .
  #<vector 08b26850>)
= #<compiled-function |ISTRING;elt;$IC;30|>
(QREFELT $ ...)

(SVREF $ ...)
$
= #<vector 08b8cc94>
64
= "ABCDEFGHJKLMNOPQRSTUVWXYZ"
= "ABCDEFGHJKLMNOPQRSTUVWXYZ"
(+ (- |i| ...) ...)

(- |i| ...)
|i|
= 10
10
= 0
(SPADCALL (QREFELT $ ...) ...)

(LET (#) ...)
(QREFELT $ ...)
(SVREF $ ...)
$
= #<vector 08b8cc94>
68
= (#<compiled-function
  |ISTRING;minIndex;$I;11|> .
  #<vector 08b26850>)
= (#<compiled-function
  |ISTRING;minIndex;$I;11|> .
  #<vector 08b26850>)
(THE (VALUES T) ...)
(FUNCALL (CAR #:G1778) ...)
(CAR #:G1778)
#:G1778
= (#<compiled-function
  |ISTRING;minIndex;$I;11|> .

```

```

      #<vector 08b26850>
    = #<compiled-function
      |ISTRING;minIndex;$I;11|>
    (QREFELT $ ...)
(SVREF $ ...)
$
      = #<vector 08b8cc94>
      64
      = "ABCDEFGHJKLMNOPQRSTUVWXYZ"
      = "ABCDEFGHJKLMNOPQRSTUVWXYZ"
      (CDR #:G1778)
#:G1778
      = (#<compiled-function
        |ISTRING;minIndex;$I;11|> .
        #<vector 08b26850>)
      = #<vector 08b26850>
      = 1
      = 1
      = 1
      = 1
      = 1
      = 1
      (CDR #:G1777)
#:G1777
      = (#<compiled-function
        |ISTRING;elt;$IC;30|> .
        #<vector 08b26850>)
      = #<vector 08b26850>
      = 65
      = 65
      = 65
      = 65
      (CDR #:G1776)
#:G1776
      = (#<compiled-function
        |CHAR;coerce;$Of;12|> .
        #<vector 08b3901c>)
      = #<vector 08b3901c>
      = #\A
      = #\A
      = #\A
      = #\A
      = #\A
      <1 (LET (# #) ...)
      (QUOTE (10 #<vector 08b8cc94>))
      (QUOTE (#\A))
      (CONS (QUOTE |RADIX;intToExpr|) ...)
      (QUOTE |RADIX;intToExpr|)
      VALUES
        = (#\A)
        = (|RADIX;intToExpr| #\A)
        = (|RADIX;intToExpr| #\A)
      (|RADIX;intToExpr| #\A)
      = #\A
      = #\A

```

```
Value = #\A
(4) ->
```

If we examine the source code for this function in `int/algebra/radix.spad` we find:

```
ALPHAS : String := "ABCDEFGHJKLMNOPQRSTUVWXYZ"

intToExpr(i:I): OUT ==
  -- computes a digit for bases between 11 and 36
  i < 10 => i :: OUT
  elt(ALPHAS, (i-10) + minIndex(ALPHAS)) :: OUT
```

We do some lookups by hand to find out what functions are being called from the domain vectors thus:

```
(4) -> )lisp (qrefelt tpd 68)
```

```
Value = (#<compiled-function
         |ISTRING;minIndex;$I;11|> . #<vector 08b26850>)
```

The #

A value appears from a call to `CHAR;coerce;$Of;12`. We can look in `CHAR.NRLIB/code.lisp` for this function and continue our descent into the code. The function looks like:

```
(DEFUN |CHAR;coerce;$Of;12| (|c| $)
  (ELT (QREFELT $ 10)
    (+ (QREFELT $ 11) (SPADCALL |c| (QREFELT $ 21))))))
```

Again we need to get the domain vector, this time from the CHAR domain. The domain vector has all of the information about a domain including what functions are referenced and what data values are used. The QREFELT is a “quick elt” function which resolved to a highly type optimized function call. The SPADCALL function funcalls the second argument to SPADCALL with the first argument to SPADCALL effectively giving:

```
(funcall (qrefelt $ 21) |c|)
```

So we modify the `CHAR;coerce;$Of;12` function to capture the domain vector thus:

```
(DEFUN |CHAR;coerce;$Of;12| (|c| $)
  (format t "|CHAR;coerce;$Of;12| called")
  (setq tpd1 $)
  (ELT (QREFELT $ 10)
    (+ (QREFELT $ 11) (SPADCALL |c| (QREFELT $ 21))))))
```

Again we rerun the failing function and now `tpd1` contains the domain vector for the domain CHAR:

## Operating system level I/O trace (strace)

If the bug seems to happen during startup the only method of debugging might be to use `strace`. To do this, replace the `$AXIOM/bin/AXIOMsys` binary with a shell script. You should:

1. rename `$AXIOM/bin/AXIOMsys` to `$AXIOM/bin/AXIOMsys.bin`
2. create the shell script shown below



3. copy the shell script to the file `$AXIOM/bin/AXIOMsys` so it will execute in place of the normal Axiom image
4. `chmod +x $AXIOM/bin/AXIOMsys` to make the script executable
5. start axiom normally

The script reads:

```
#!/bin/sh
exec strace -o /tmp/str.$$ /research/test/mnt/ubuntu/bin/AXIOMsys.bin "$@"
  | tee /tmp/tee.$$
```

The script will create 2 files in the tmp directory, “str.NNNNN” and “tee.NNNNN” where NNNNN is the process id assigned to axiom at runtime.

The tee.NNNNN file contains the console output you saw. The str.NNNNN contains the output of strace which is a list of all of the system calls and their result.

## 0.23 How to make graphs in algebra books

```
dot -Tps |pic >books/ps/domain.ps
```

where file pic contains something like:

```
digraph pic {
  fontsize=10;
  bgcolor="#ECEA81";
  node [shape=box, color=white, style=filled];

  "OrderlyDifferentialVariable"
  [color=lightblue,href="bookvol10.3.pdf#nameddest=ODVAR"];

}
```

In book Volume 10.3 there are .ps files generated for each domain.

These pictures show the category, domain, or package that is the highest in the algebra tower. That means that the parent domain has to be compiled before this domain. Since the parent is the highest in the tower then all of the other domains will already have been compiled.

We derive the dependency information from the algebra Makefile. In order to insert algebra into the tower we derive its parents and then create a table of all of the parents. However, the table is mostly comments except for the highest parent. So if the highest parent is compiled in layer 12 then the new domain is inserted into layer 13.

The graph information at the end of a domain (e.g. UTSZ) contains only the uncommented information. It is used to generate the picture. So for UTSZ we see:

```
"UTSZ" [color="#88FF44",href="bookvol10.3.pdf#nameddest=UTSZ"]
"ACFS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACFS"]
"UTSZ" -> "ACFS"
```

This tells us several pieces of information. UTSZ depends on ACFS as the highest parent. ACFS is in layer 17 so is in layer 18. ACFS is a category (color 4488FF) and lives in bookvol10.2. UTSZ is a domain (color 88FF44, note the rotation of hex codes, a package is color FF4488) and lives in bookvol10.3.

The necessary stanzas exist after each algebra domain. Copy the stanza into a digraph block and run the dot function. So for the UTSZ domain we would create a file (e.g. called 'pic') that contains a block that reads:

```
digraph pic {
  fontsize=10;
  bgcolor="#ECEA81";
  node [shape=box, color=white, style=filled];

  "UTSZ" [color="#88FF44",href="bookvol10.3.pdf#nameddest=UTSZ"]
  "ACFS" [color="#4488FF",href="bookvol10.2.pdf#nameddest=ACFS"]
  "UTSZ" -> "ACFS"

}
```

and then run the command:

```
dot -Tps <pic >pic.ps
cp pic.ps ps/v103univariatetaylorseriesczero.ps
```

The output file "ps/v103univariatetaylorseriesczero.ps" is included after the pagepic of each domain in the book.

## 0.24 Adding or Editing pages in Hyperdoc

In Axiom it is easy to develop new pages in hyperdoc. All of the hyperdoc pages live in bookvol7.1.pamphlet.

The structure of each page, say for the Fantastic domain, consists of a few lines of latex followed by a literate chunk. All of the hyperdoc page information goes into the literate chunk and will be extracted as a hyperdoc page.

```
\section{fantastic.ht} <-- ordinary latex
\pagehead{FantasticPage}{fantastic.ht}{Fantastic} <-- special latex
\pageto{....} <-- for latex (not hyperdoc) links to other pages
<<fantastic.ht>>= <-- this is what htadd looks for

  all the documentation for domain Fantastic

@ <-- this is the end of the page for htadd
```

When you add new pages to bookvol7.1.pamphlet you need to tell the system about the changes. The "htadd" function will search the bookvol7.1.pamphlet file for chunks with the name "\*.ht" and build the file ht.db.

The ht.db file is used by hyperdoc to find pages. Each line in ht.db contains a line that

```
FantasticPage (byteIndex) (lineIndex)
```

So the two critical files, bookvol7.1.pamphlet and ht.db live in \$AXIOM/doc

There is a very fast cycle for editing.

```
cd $AXIOM/doc
```

```
while (1) do
  axiom <-- at the shell prompt
  (navigate to page in hyperdoc) <-- to check your work
```

```

)lisp (bye)                <-- at the axiom prompt
(modify bookvol7.1.pamphlet) <-- change the page
rm ht.db                   <-- remove the old database
htadd bookvol7.1.pamphlet  <-- remake the database

```

## 0.25 Graphviz file creation

The graphviz output used on the website is a scaled vector graphics file (SVG). The dot command to output this file is:

```
dot -Tsvg:cg <pic >pic.svg
```

The SVG file that gets generated has the following preamble.

```

<?xml version="1.0" encoding="UTF-8" standalone="no"?>
<!DOCTYPE svg PUBLIC "-//W3C//DTD SVG 1.0//EN"
"http://www.w3.org/TR/2001/REC-SVG-20010904/DTD/svg10.dtd" [
<!-- ATTLIST svg xmlns:xlink CDATA #FIXED "http://www.w3.org/1999/xlink">
]>
<!-- Generated by dot version 2.8 (Thu Sep 14 20:34:11 UTC 2006)
For user: (root) root -->
<!-- Title: AxiomSept2008 Pages: 1 -->
<svg width="3960pt" height="2312pt"
viewBox = "0 0 3960 2312"
xmlns="http://www.w3.org/2000/svg"
xmlns:xlink="http://www.w3.org/1999/xlink">

```

There are two pieces of information that are important. First, we need to add the following text by removing the trailing > character from the svg tag and replacing it with the following block. This block exports some javascript functions that we use to scale the graphics.

```

onload="RunScript(evt)">
<script type="text/ecmascript">
<![CDATA[
var g_element;
var SVGDoc;
var SVGRoot;
function setDimension(w,h) {
    SVGDoc.documentElement.setAttribute("width",w);
    SVGDoc.documentElement.setAttribute("height",h);
}
function setScale(sw,sh) {
    g_element.setAttribute("transform","scale("+sw+" "+sh+)");
}
function RunScript(LoadEvent) {
    top.SVGsetDimension=setDimension;
    top.SVGsetScale=setScale;
    SVGDoc=LoadEvent.target.ownerDocument;
    g_element=SVGDoc.getElementById("graph0");
}
]]>
</script>

```

A second item of interest is the viewbox line which gives us the width and height information. We use this information to place the graph on the web page. A simple example of the web

page looks follows. We need to replace the X and Y sizes with the sizes from the viewBox above.

```

<html>
<head>
<title>Axiom Abbreviated Category and Domain graph</title>
<script type="text/javascript">
var W3CDOM = (document.createElement && document.getElementsByTagName);
window.onload = init;
function init(evt) {
SVGscale(0.5);
}
function SVGscale(scale) {
window.SVGsetDimension(8162*scale, 3068*scale);
window.SVGsetScale(scale,scale);
var box = document.getElementById('svgid');
box.width = 8162*scale;
box.height = 3068*scale;
}
</script>

</head>
<body>
<h1>Axiom Abbreviated Category and Domain graph</h1>
<div>
choose here:
<a href="#" onclick="SVGscale(0.1);">0.1</a> or
<a href="#" onclick="SVGscale(0.25);">0.25</a> or
<a href="#" onclick="SVGscale(0.5);">0.5</a> or
<a href="#" onclick="SVGscale(1);">1.0</a> or
<a href="#" onclick="SVGscale(1.5);">1.5</a> or ...
</div>
<div>
  <object id='svgid' data="dotabb.svg" type="image/svg+xml"
    width="8162" height="3068" wmode="transparent" style="overflow:hidden;" />
  </object>
</div>

</body>
</html>

```

## 0.26 Adding Algebra

### Adding algebra to the books

Assume you have a piece of algebra code that you want to permanently add to Axiom. This is a fairly complex process since the system automatically runs regression tests, creates help files, etc. and has other standard features you need to support. Lets assume your algebra looks like this:

```

)abbreviation package INTERGB InterfaceGroebnerPackage
InterfaceGroebnerPackage(K,symb,E,OV,R):Exports == Implementation where
  K      : FIELD

```

```

symb : List Symbol
E     : OrderedAbelianMonoidSup
OV    : OrderedSet
R     : PolynomialCategory(K,E,OV)

LIST ==> List

Exports ==> with
  groebner: LIST R -> LIST R

Implementation ==> add
  PF ==> PrimeField(q)
  DPF ==> DistributedMultivariatePolynomial(symb,PF)
  D ==> DistributedMultivariatePolynomial(symb,K)
  JCFGBPack ==> GroebnerPackage(PF,E,OV,DPF)
  GBPack ==> GroebnerPackage(K,E,OV,D)

coerceDtoR: D->R
coerceDtoR(pol) == map(#1,pol)$MPolyCatFunctions2(OV,E,E,K,K,D,R)

groebner(l)==
  ldmp:List D:= [coerceRtoD(pol) for pol in l]
  gg:=groebner(ldmp)$GBPack
  [coerceDtoR(pol) for pol in gg]

```

There are some things to check and things to change.

- remove all tabs. Spad is a language that assigns meaning to indentation and tabs are not going to survive the build process intact.
- try to stay within 80 characters. Spad code is printed in the books so it should try to limit line lengths everywhere.
- change “)abbreviation” to “)abbrev”. The Makefile will search for the abbrev line and expect this exact text.

```
)abbrev package INTERGB InterfaceGroebnerPackage
```

- make sure there is only a single space between the items in the abbrev line. The Makefile assumes this.
- Add the comment header block. The author information is used to check that all authors are included in the credits. The description tag is used as output by the “)describe” command. For example:

```

++ Author: Gaetan Hache
++ Date Created: September 1996
++ Date Last Updated: April, 2010, by Tim Daly
++ Description:
++ Part of the Package for Algebraic Function Fields in one variable PAFF

```

Other tags can be included but are not used. Do not assume that any format information will be correctly translated or preserved. Make the description section be simple text with a single character between the “++” and the first word of the text.

1. Choose the right book:
  - Category goes into book bookvol10.2.pamphlet

- Domain goes into book bookvol10.3.pamphlet
  - Package goes into book bookvol10.4.pamphlet
  - Numerics goes into book bookvol10.5.pamphlet
2. Find the right chapter. Chapter ordered by name, not abbreviation
  3. Create a new section. The easiest way to do this is to copy another section and change the names. In any case, your new section will need
  4. Create a dividing line consisting of all % signs. Note that this character is the comment character for TeX.
  5. Create the `\section` tag. This tag should contain the exact text of the `)abbrev` line from your algebra code. It reads: `)section{package INTERGB InterfaceGroebnerPackage}`
  6. Create a regression test chunk. This chunk derives its name from the algebra name as in “InterfaceGroebnerPackage.input”. This chunk will be automatically extracted and run during regression testing. For the moment the chunk should be a simple empty input file as in:

```
)set break resume
)sys rm -f InterfaceGroebnerPackage.output
)spool InterfaceGroebnerPackage.output
)set message test on
)set message auto off
)clear all
```

```
--S 1 of 1
)show InterfaceGroebnerPackage
--E 1
```

```
)spool
)lisp (bye)
```

7. Create a help text chunk. This chunk derives its name from the algebra name as in “InterfaceGroebnerPackage.help”. This chunk will be automatically extracted and used to build help text during build. For the moment the chunk should be a simple empty help text file as in:

```
=====
examples InterfaceGroebnerPackage
=====
```

```
See Also:
o )show InterfaceGroebnerPackage
```

At this point we need information from the Axiom interpreter to continue. Start Axiom and compile your program with “autoload” on. For example:

```
axiom -nox
)set message autoload on
)co InterfaceGroebnerPackage.spad
```

You will see output containing lines which detail the category, domains, and packages needed by your code, for example:

```
Loading /research/test/mnt/ubuntu/algebra/FIELD.o for category Field
```

Loading /research/test/mnt/ubuntu/algebra/EUCDDOM.o for category  
 EuclideanDomain  
 Loading /research/test/mnt/ubuntu/algebra/PID.o for category  
 PrincipalIdealDomain  
 Loading /research/test/mnt/ubuntu/algebra/GCDDOM.o for category  
 GcdDomain  
 Loading /research/test/mnt/ubuntu/algebra/INTDOM.o for category  
 IntegralDomain  
 Loading /research/test/mnt/ubuntu/algebra/COMRING.o for category  
 CommutativeRing  
 Loading /research/test/mnt/ubuntu/algebra/RING.o for category Ring  
 Loading /research/test/mnt/ubuntu/algebra/RNG.o for category Rng  
 Loading /research/test/mnt/ubuntu/algebra/ABELGRP.o for category  
 AbelianGroup  
 Loading /research/test/mnt/ubuntu/algebra/CABMON.o for category  
 CancellationAbelianMonoid  
 Loading /research/test/mnt/ubuntu/algebra/ABELMON.o for category  
 AbelianMonoid  
 Loading /research/test/mnt/ubuntu/algebra/ABELSG.o for category  
 AbelianSemiGroup  
 Loading /research/test/mnt/ubuntu/algebra/SETCAT.o for category  
 SetCategory  
 Loading /research/test/mnt/ubuntu/algebra/BATYPE.o for category  
 BasicType  
 Loading /research/test/mnt/ubuntu/algebra/KOERCE.o for category  
 CoercibleTo  
 Loading /research/test/mnt/ubuntu/algebra/SGROUP.o for category  
 SemiGroup  
 Loading /research/test/mnt/ubuntu/algebra/MONOID.o for category  
 Monoid  
 Loading /research/test/mnt/ubuntu/algebra/LMODULE.o for category  
 LeftModule  
 Loading /research/test/mnt/ubuntu/algebra/BMODULE.o for category  
 BiModule  
 Loading /research/test/mnt/ubuntu/algebra/RMODULE.o for category  
 RightModule  
 Loading /research/test/mnt/ubuntu/algebra/ALGEBRA.o for category  
 Algebra  
 Loading /research/test/mnt/ubuntu/algebra/MODULE.o for category  
 Module  
 Loading /research/test/mnt/ubuntu/algebra/ENTIRER.o for category  
 EntireRing  
 Loading /research/test/mnt/ubuntu/algebra/UFD.o for category  
 UniqueFactorizationDomain  
 Loading /research/test/mnt/ubuntu/algebra/DIVRING.o for category  
 DivisionRing  
 Loading /research/test/mnt/ubuntu/algebra/OAMONS.o for category  
 OrderedAbelianMonoidSup  
 Loading /research/test/mnt/ubuntu/algebra/OCAMON.o for category  
 OrderedCancellationAbelianMonoid  
 Loading /research/test/mnt/ubuntu/algebra/OAMON.o for category  
 OrderedAbelianMonoid  
 Loading /research/test/mnt/ubuntu/algebra/OASGP.o for category

```

OrderedAbelianSemiGroup
Loading /research/test/mnt/ubuntu/algebra/ORDSET.o for category
OrderedSet
Loading /research/test/mnt/ubuntu/algebra/POLYCAT.o for category
PolynomialCategory
Loading /research/test/mnt/ubuntu/algebra/PDRING.o for category
PartialDifferentialRing
Loading /research/test/mnt/ubuntu/algebra/FAMR.o for category
FiniteAbelianMonoidRing
Loading /research/test/mnt/ubuntu/algebra/AMR.o for category
AbelianMonoidRing
Loading /research/test/mnt/ubuntu/algebra/CHARZ.o for category
CharacteristicZero
Loading /research/test/mnt/ubuntu/algebra/CHARNZ.o for category
CharacteristicNonZero
Loading /research/test/mnt/ubuntu/algebra/FRETRACT.o for category
FullyRetractableTo
Loading /research/test/mnt/ubuntu/algebra/RETRACT.o for category
RetractableTo
Loading /research/test/mnt/ubuntu/algebra/EVALAB.o for category
Evalable
Loading /research/test/mnt/ubuntu/algebra/IEVALAB.o for category
InnerEvalable
Loading /research/test/mnt/ubuntu/algebra/FLINEXP.o for category
FullyLinearlyExplicitRingOver
Loading /research/test/mnt/ubuntu/algebra/LINEXP.o for category
LinearlyExplicitRingOver
Loading /research/test/mnt/ubuntu/algebra/KONVERT.o for category
ConvertibleTo
Loading /research/test/mnt/ubuntu/algebra/PATMAB.o for category
PatternMatchable
Loading /research/test/mnt/ubuntu/algebra/PFECAT.o for category
PolynomialFactorizationExplicit
Loading /research/test/mnt/ubuntu/algebra/FFIELDC.o for category
FiniteFieldCategory
Loading /research/test/mnt/ubuntu/algebra/FPC.o for category
FieldOfPrimeCharacteristic
Loading /research/test/mnt/ubuntu/algebra/FINITE.o for category
Finite
Loading /research/test/mnt/ubuntu/algebra/STEP.o for category
StepThrough
Loading /research/test/mnt/ubuntu/algebra/DIFRING.o for category
DifferentialRing

Loading /research/test/mnt/ubuntu/algebra/NNI.o for domain
NonNegativeInteger
Loading /research/test/mnt/ubuntu/algebra/INT.o for domain Integer

```

These are all of the algebra files on which your algebra depends. Collect all of these names into a list:

```

FIELD.o      EUCDOM.o    PID.o       GCDDOM.o    INTDOM.o
COMRING.o   RING.o      RNG.o       ABELGRP.o   CABMON.o
ABELMON.o   ABELSG.o   SETCAT.o   BASTYPE.o   KOERCE.o
SGROUP.o    MONOID.o    LMODULE.o  BMODULE.o   RMODULE.o

```



|           |           |           |          |           |
|-----------|-----------|-----------|----------|-----------|
| ALGEBRA.o | MODULE.o  | ENTIRER.o | UFD.o    | DIVRING.o |
| OAMONS.o  | OCAMON.o  | OAMON.o   | OASGP.o  | ORDSET.o  |
| POLYCAT.o | PDRING.o  | FAMR.o    | AMR.o    | CHARZ.o   |
| CHARNZ.o  | FRETRCT.o | RETRACT.o | EVALAB.o | IEVALAB.o |
| FLINEXP.o | LINEXP.o  | KONVERT.o | PATMAB.o | PFECAT.o  |
| FFIELDC.o | FPC.o     | FINITE.o  | STEP.o   | DIFRING.o |
| NNI.o     | INT.o     |           |          |           |

The algebra files are arranged into layers. Algebra in a given layer can only depend on algebra in lower layers. So algebra in layer 4 can only depend on algebra in layers 0 through 3.

Your algebra depends on all of the above algebra so we first need to answer the question “What is the highest layer of algebra my code depends upon”.

To answer that question we need to know the layer of each of the above files. This can be determined by searching the Makefile and finding the layer. We do this here and annotate the above list, rearranged by layers:

|           |           |           |           |           |
|-----------|-----------|-----------|-----------|-----------|
| FIELD.o   | EUCDOM.o  | PID.o     | GCDDOM.o  | INTDOM.o  |
| COMRING.o | RING.o    | RNG.o     | ABELGRP.o | CABMON.o  |
| ABELMON.o | ABELSG.o  | SETCAT.o  | BASTYPE.o | KOERCE.o  |
| SGROUP.o  | MONOID.o  | LMODULE.o | BMODULE.o | RMODULE.o |
| ALGEBRA.o | MODULE.o  | ENTIRER.o | UFD.o     | DIVRING.o |
| OAMONS.o  | OCAMON.o  | OAMON.o   | OASGP.o   | ORDSET.o  |
| POLYCAT.o | PDRING.o  | FAMR.o    | AMR.o     | CHARZ.o   |
| CHARNZ.o  | FRETRCT.o | RETRACT.o | EVALAB.o  | IEVALAB.o |
| FLINEXP.o | LINEXP.o  | KONVERT.o | PATMAB.o  | PFECAT.o  |
| FFIELDC.o | FPC.o     | FINITE.o  | STEP.o    | DIFRING.o |
| NNI.o     | INT.o     |           |           |           |

We find that

layer 0

EUCDOM.o GCDDOM.o INTDOM.o COMRING.o RING.o RNG.o ABELGRP.o CABMON.o ABELMON.o  
 ABELSG.o SETCAT.o BASTYPE.o KOERCE.o MONOID.o ENTIRER.o UFD.o DIVRING.o  
 POLYCAT.o KONVERT.o FFIELDC.o DIFRING.o NNI.o INT.o

layer 1

SGROUP.o LMODULE.o RMODULE.o ORDSET.o RETRACT.o IEVALAB.o FINITE.o STEP.o  
 PATMAB.o

layer 2

BMODULE.o OASGP.o PDRING.o CHARZ.o CHARNZ.o EVALAB.o LINEXP.o

layer 3

MODULE.o OAMON.o

layer 4

ALGEBRA.o OCAMON.o

layer 5

PID.o OAMONS.o

layer 6

FIELD.o AMR.o FRETRCT.o FLINEXP.o

```
layer 7
FAMR.o FPC.o
```

```
layer 10
PFECAT.o
```

So we know that our algebra belongs in layer 11 since it depends on PFECAT which lives in layer 10.

Now we have all of the information needed to add the algebra to the book. We create a new section that contains the following parts:

- the `\section{...}` header
- the input file
- the help file
- the page head markup
- the pagepic markup
- the Exports section
- the theory and discussion section
- the algebra
- the dot picture file information

We've already explained most of the sections. Now we insert a chunk for our algebra. The name of this chunk is important because the Makefile will look for this exact chunk name. In our example case it looks like:

```
\begin{chunk}{package INTERGB InterfaceGroebnerPackage}
```

which is followed immediately by the algebra starting with the “abbrev” command.

Make sure that the section names, input file names, help file names, and chunk names all reflect the new algebra names.

Next we copy the chunkname to the bottom of the file as part of the algebra chunk.

The next piece to be handled is the “pagepic” tag. This inserts a picture into the book which shows what files our algebra depends upon. We do this using the graphviz dot program.

We create a file that shows the graph relationship. Since our code INTERGB depends on PFECAT we show that here. We create a line for our code that looks like:

```
"INTERGB" [color="#FF4488",href="bookvol10.4.pdf#nameddest=INTERGB"]
```

This tells us the name of the node, the background color where

- #4488FF is a category
- #88FF44 is a domain
- #FF4488 is a package
- #FF8844 is a numeric

(note the rotation of bytes) and the html link from the graph to the position in the document so the user can click on the picture and go to the source code. It names the book and the offset.

We have the same information for PFECAT:

```
"PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
```

We combine this information to create a picture file for the graphviz dot program:

```
digraph pic {
  fontsize=10;
  bgcolor="#ECEA81";
  node [shape=box, color=white, style=filled];

  "INTERGB" [color="#FF4488",href="bookvol10.4.pdf#nameddest=INTERGB"]
  "PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
  "INTERGB" -> "PFECAT"

}
```

Save this file into `$AXIOM/books/pic`. Then run:

```
dot -Tps <pic >ps/v104interfacegroebnerpackage.ps
```

This will generate a postscript file `$AXIOM/books/ps/v104interfacegroebnerpackage.ps`

Note that the pagepic name should be lower case as this is the name of a file in the file system. Axiom uses only lower case names as files to avoid the problem of mixed-case or ambiguous case file systems.

The pagepic file is saved into a subdirectory (ps) under the books directory. The file contains the name of the book (v104 is volume 10.4), the name of the algebra in lowercase, and the extension of ps.

Now we modify the pagepic tag in book volume 10.4 to read:

```
\pagepic{ps/v104interfacegroebnerpackage.ps}{INTERGB}{1.00}
```

This tells us where to find the file, what the abbreviation for the file will be, and the scale factor to use to display the file (1.00).

We copy the body of the pic file to the book so we can easily recreate the graphs:

```
\begin{chunk}{INTERGB.dotabb}
  "INTERGB" [color="#FF4488",href="bookvol10.4.pdf#nameddest=INTERGB"]
  "PFECAT" [color="#4488FF",href="bookvol10.2.pdf#nameddest=PFECAT"]
  "INTERGB" -> "PFECAT"
\end{chunk}
```

Next we return to the command line and get explicit information about our new domain:

```
(1) -> )show INTERGB
InterfaceGroebnerPackage(K: Field,
                        symb: List Symbol,
                        E: OrderedAbelianMonoidSup,
                        OV: OrderedSet,
                        R: PolynomialCategory(K,E,OV))
      is a package constructor
Abbreviation for InterfaceGroebnerPackage is INTERGB
This constructor is exposed in this frame.
Issue )edit /research/silver/PAFF/PAFF/spad/interGBwoGB.spad to see
      algebra source code for INTERGB
```

```
----- Operations -----
groebner : List R -> List R
```

We need to insert the results of this command into the trivial input file we have created. The trivial input file will be run during build time and the results of the command at build time will be compared with the results we provide. The results we provide are actually comments so they begin with two dashes, the Axiom comment character.

The regression program (“regress”) compares each line generated by the program with the line stored in the input file. It only compares lines that start with “-R”. Each test is surrounded by the comment pair “-S m of n” and “-E m” which indicates the start and end of test “m” respectively.

Since we will eventually compile this domain from the book the final output line for the )edit location will be the book. Thus we need to change the line:

```
Issue )edit /PAFF/interGBwoGB.spad to see algebra source code for INTERGB
```

to read:

```
Issue )edit bookvol10.4.pamphlet to see algebra source code for INTERGB
```

So we update our input file section to read:

```
)set break resume
)sys rm -f InterfaceGroebnerPackage.output
)spool InterfaceGroebnerPackage.output
)set message test on
)set message auto off
)clear all

--S 1 of 1
)show InterfaceGroebnerPackage
--R InterfaceGroebnerPackage(K: Field,
--R      symb: List Symbol,
--R      E: OrderedAbelianMonoidSup,
--R      OV: OrderedSet,
--R      R: PolynomialCategory(K,E,OV))
--R   is a package constructor
--R Abbreviation for InterfaceGroebnerPackage is INTERGB
--R This constructor is exposed in this frame.
--R Issue )edit bookvol10.4.pamphlet to see algebra source code for INTERGB
--R
--R----- Operations -----
--R groebner : List R -> List R
--R
--E 1

)spool
)lisp (bye)
```

There is another section, Exports, which lists all of the unique function names exported by this algebra. Note that this list is longer than just the functions locally defined because the algebra inherits functions. We make a tabular environment with the number of columns specified by the “l” characters (“l” means left-justify, “c” would be center, and “r” would be right justify). We include just enough columns to keep from overflowing the 80 column limit.

For this domain we see generate we only have a single export.

```
{\bf Exports:}\
\cross{INTERGB}{groebner}
```

The cross function creates a cross-reference entry in the index so you can look up the function and find the domain or look up the domain and find the function.

Now we have finished setting up the algebra.

We have to add the algebra, regression, and help information to the Makefile.

We know from above that the algebra belongs in layer 11 so we find layer 11 and add this abbreviation in alphabetical order. We can use this order since none of the files depend on each other in this layer. The new line looks like:

```
 ${OUT}/INMODGCD.o ${OUT}/INNMFAC.o ${OUT}/INPSIGN.o ${OUT}/INTERGB.o \
```

Note that the trailing slash is required by the Makefile in order to continue the input line.

Take note of the prior algebra abbreviation INPSIGN. We search for that lower down and find the block reading:

```
"INPSIGN" [color="#FF4488",href="bookvol10.4.pdf#nameddest=INPSIGN"]
/*"INPSIGN" -> {"RING"; "RNG"; "ABELGRP"; "CABMON"; "ABELMON"; "ABELSG"}*/
/*"INPSIGN" -> {"SETCAT"; "BASTYPE"; "KOERCE"; "SGROUP"; "MONOID"}*/
/*"INPSIGN" -> {"LMODULE"; "UPOLYC"; "POLYCAT"; "PDRING"; "FAMR"; "AMR"}*/
/*"INPSIGN" -> {"BMODULE"; "RMODULE"; "COMRING"; "ALGEBRA"; "MODULE"}*/
/*"INPSIGN" -> {"CHARZ"; "CHARNZ"; "INTDOM"; "ENTIRER"; "FRETRCT"}*/
/*"INPSIGN" -> {"RETRACT"; "EVALAB"; "IEVALAB"; "FLINEXP"; "LINEXP"}*/
/*"INPSIGN" -> {"ORDSET"; "KONVERT"; "PATMAB"; "GCDDOM"}*/
"INPSIGN" -> "PFECAT"
/*"INPSIGN" -> {"UFD"; "ELTAB"; "DIFRING"; "DIFEXT"; "STEP"; "EUCDOM"}*/
/*"INPSIGN" -> {"PID"; "FIELD"; "DIVRING"; "INT"; "INS-"}*/
```

This information is used to create the total graph of the algebra. Most of this information is kept for future graph use but commented out. We only uncomment lines that are direct dependence relationships. As you can see from the above INPSIGN depends only on PFECAT.

First we create the signature line which is the same one used above for the pagepic graph.

We also need to create a similar block for our code. We use the list generated above, replacing the “.o” with semicolons.

The total result is:

```
"INTERGB" [color="#FF4488",href="bookvol10.4.pdf#nameddest=INTERGB"]
/*"INTERGB" -> {
/*"INTERGB" -> {"FIELD"; "EUCDOM"; "PID"; "GCDDOM"; "INTDOM"; "COMRING"}*/
/*"INTERGB" -> {"RING"; "RNG"; "ABELGRP"; "CABMON"; "ABELMON"; "ABELSG"}*/
/*"INTERGB" -> {"SETCAT"; "BASTYPE"; "KOERCE"; "SGROUP"; "MONOID"}*/
/*"INTERGB" -> {"LMODULE"; "BMODULE"; "RMODULE"; "ALGEBRA"; "MODULE"}*/
/*"INTERGB" -> {"ENTIRER"; "UFD"; "DIVRING"; "OAMONS"; "OCAMON"; "OAMON"}*/
/*"INTERGB" -> {"OASGP"; "ORDSET"; "POLYCAT"; "PDRING"; "FAMR"; "AMR"}*/
/*"INTERGB" -> {"CHARZ"; "CHARNZ"; "FRETRCT"; "RETRACT"; "EVALAB"}*/
/*"INTERGB" -> {"IEVALAB"; "FLINEXP"; "LINEXP"; "KONVERT"; "PATMAB"}*/
"INTERGB" -> "PFECAT"
/*"INTERGB" -> {"FFIELD"; "FPC"; "FINITE"; "STEP"; "DIFRING"; "NNI"; "INT"}*/
```

Help files are automatically extracted from the books using the lisp function “makeHelpFiles” which lives in books/tangle.lisp. This will find all of the .help chunks in books of interest and write each chunk to the target directory in its own filename. So if a chunk name is

somedomain.help we create the help file somedomain.help containing the chunk value.

Now we create the regression hook. Look for the list REGRESS and add the line containing the algebra thus:

```
InterfaceGroebnerPackage.regress \
```

This will cause regression to be run on the InterfaceGroebnerPackage.input file.

We need to modify book volume 5 (the interpreter) to add this algebra to the list of exposed algebra. To do this, add the line:

```
(|InterfaceGroebnerPackage| . INTERGB)
```

to the variable \$globalExposureGroupAlist under the “basic” sublist.

Now the algebra should build and have the new algebra available. There are ways this can fail which we will cover in more detail.

Once the code compiles cleanly there are a few ways to check that it works. First, check that the file int/input/InterfaceGroebnerPackage.regress shows no failures. Second, run the command

```
)show InterfaceGroebnerPackage
```

to see if the domain exists. Run the command

```
)help InterfaceGroebnerPackage
```

to see that the help file exists. Run the command

```
)describe InterfaceGroebnerPackage
```

to see that the description paragraph exists.

## Creating a stand-alone pamphlet file

Suppose we want to add a new algebra file from a new pamphlet. For example, we want to add BLAS1.spad in books/newbook.pamphlet. The explanation for the steps follow. The steps are:

1. write the algebra code
2. create the pamphlet file called newbook.pamphlet
3. create a new section in newbook.pamphlet
4. create the chunk name stanza in newbook.pamphlet
5. insert BLAS1.spad into the new stanza
6. create the dotabb stanza in newbook.pamphlet
7. update src/algebra/Makefile.pamphlet to insert BLAS1.spad
8. update src/algebra/Makefile.pamphlet to insert BLAS1 into graph
9. update the dotabb stanza in newbook.pamphlet
10. update src/Makefile.pamphlet to copy the newbook file to src/algebra
11. update \$globalExposureGroupAlist in bookvol5 to add BLAS1 to basic

## 0.27 Makefile

This book is actually a literate program [Knut92] and can contain executable source code. In particular, the Makefile for this book is part of the source of the book and is included below.

# Bibliography

[Bake14] Martin Baker. Axiom architecture, 2014.

**Link:** [http://www.euclideanspace.com/prog/scratchpad/  
internals/ccode](http://www.euclideanspace.com/prog/scratchpad/internals/ccode)

[Knut92] Donald E. Knuth. *Literate Programming*. Center for the Study of Language and Information, Stanford CA, 1992, 0-937073-81-4.



